Math 2233.21569
FIRST EXAM
February 23, 2021

Name:_____

1. (15 pts) Classify the following differential equations: determine their order, if they are linear or non-linear, and if they are ordinary differential equations or partial differential equations.

(a) $\dfrac{\partial^2 \phi}{\partial t^2} - x^2 \dfrac{\partial^2 \phi}{\partial x^2} = 0$

(b) $\dfrac{dy}{dx} + x^2 y = e^x$

(c) $\dfrac{d^2 x}{dt^2} + x\dfrac{dx}{dt} + x = 0$

(d) $\dfrac{d^3 \phi}{dt^3} + t\dfrac{d\phi}{dt} = \sin(t)$

(e) $\dfrac{\partial^2 \phi}{\partial x^2} + y \left(\dfrac{\partial \phi}{\partial y}\right)^2 = 0$

2. (15 pts) Consider the following first order ODE: $y' = xy + y$ and suppose $y(x)$ is the solution satisfying $y(0) = 1$. Use the numerical (Euler) method with $n = 3$ and $\Delta x = 0.1$ to estimate $y(0.3)$.

3. (15 pts) Find an explicit solution of the following (separable) differential equation.

$$x + \sin(y) \, y' = 2$$

4. (15 pts) Solve the following initial value problem

$$xy' - y = x \quad , \quad y(1) = 0$$

5. (20 pts) Consider the following initial value problem

$$y + (x + 1)\frac{dy}{dx} = x$$
$$y(0) = 1$$

Show that the differential equation is exact and then find the explicit solution satisfying the initial condition.

6. (20 pts) Find the general solution of

$$x^2\frac{dy}{dx} = xy - y^2$$

(Hint: recast the differential equation into the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ and use the change of variable $z(x) = \frac{y(x)}{x}$.)