Math 2233.21570
SECOND EXAM
April 1, 2021

Name:______________________________

1. (5 pts) Explain in words and formulas how you would construct the general solution of $y'' + p(x)\,y' + q(x)\,y = g(x)$, given that $y_1(x)$ is a solution of $y'' + p(x)\,y' + q(x)\,y = 0$. (That is, describe the general procedure, writing down the relevant formulas. It is **not** necessary to carry out any calculations.)

2. (10 pts) Given that $y_1(x) = x^{-2}$ is one solution of $x^2y'' + 5xy' + 4y = 0$, use Reduction of Order to determine the general solution of this differential equation.

3. Determine the general solution of the following differential equations.

(a) (5 pts) $x^2y'' + 5xy' - 12y = 0$

(b) (5 pts) $y'' - 8y' + 16y = 0.$

(c) (5 pts) $y'' - 2y' + 5y = 0$

(d) (5 pts) $x^2y'' - 3xy' + 5y = 0$

(e) (5 pts) $x^2y'' - 3xy' + 4y = 0$

(f) (5 pts) $y'' + 9y = 0$

4. Given that $y_1(x) = e^x$ and $y_2(x) = e^{-2x}$ are solutions of $y'' + y' - 2y = 0$:

(a) (10 pts) Use the Method of Variation of Parameters to find a particular solution of $y'' + y' - 2y = 12e^{2x}$.

(b) (10 pts) Find the solution of the differential equation in part (a) satisfying $y(0) = 6$, $y'(0) = 3$.

5. Invert the following Laplace Transforms (i.e find the function $f(t)$ with the given Laplace transform).

(a) (10 pts) $\mathcal{L}[f](s) = \dfrac{s-3}{s^2-3s+2}$

(b) (10 pts) $\mathcal{L}[f](s) = \dfrac{1}{s^2+2s+10}$ (Hint: try completing the square in the dominator)

6. (15 pts) Solve the following initial value problems **using the Laplace transform method**.

$$y'' + 3y' - 4y = 0 \qquad ; \qquad y(0) = 2 \quad , \quad y'(0) = 2$$

Table of Laplace Transforms

(1) $\mathcal{L}\left[t^n\right] = \dfrac{n!}{s^{n+1}}$

(2) $\mathcal{L}\left[e^{at}\right] = \dfrac{1}{s-a}$

(3) $\mathcal{L}\left[\sin(at)\right] = \dfrac{a}{s^2+a^2}$

(4) $\mathcal{L}\left[\cos(at)\right] = \dfrac{s}{s^2+a^2}$

(5) $\mathcal{L}\left[\sinh(at)\right] = \dfrac{a}{s^2-a^2}$

(6) $\mathcal{L}\left[\cosh(at)\right] = \dfrac{s}{s^2-a^2}$

(7) $\mathcal{L}\left[e^{at}\sin(bt)\right] = \dfrac{b}{(s-a)^2+b^2}$

(8) $\mathcal{L}\left[e^{at}\cos(bt)\right] = \dfrac{s-a}{(s-a)^2+b^2}$

(9) $\mathcal{L}\left[t^n e^{at}\right] = \dfrac{n!}{(s-a)^{n+1}}$

(10) $\mathcal{L}\left[e^{at}\sinh(bt)\right] = \dfrac{b}{(s-a)^2-b^2}$

(11) $\mathcal{L}\left[e^{at}\cosh(bt)\right] = \dfrac{s-a}{(s-a)^2-b^2}$

(12) $\mathcal{L}\left[\dfrac{df}{dx}\right] = s\mathcal{C}\left[f\right] - f(0)$

(13) $\mathcal{L}\left[\dfrac{d^2f}{dx^2}\right] = s^2\mathcal{C}\left[f\right] - sf(0) - \dfrac{df}{dx}(0)$