

**Math 2233.21569**  
**SOLUTIONS TO FIRST EXAM**  
 February 23, 2021

1.

- (a) 2nd Order, Linear, PDE
- (b) 1st Order, Linear, ODE
- (c) 2nd Order, Nonlinear, ODE
- (d) 3rd Order, Linear, ODE
- (e) 2nd Order, Nonlinear, PDE

2.

$$\begin{aligned}y_1 &= y_0 + F(x_0, y_0) \Delta x = 1 + (x_0 y_0 + y_0) (0.1) = 1 + (0 + 1) (0.1) = 1.1 \\y_2 &= y_1 + (x_1 y_1 + y_1) \Delta x = 1.1 + ((0.1)(1.1) + 1.1) (0.1) = 1.221 \\y_3 &= y_2 + (x_2 y_2 + y_2) \Delta x = 1.221 + ((0.2)(1.221) + 1.221) (0.1) = 1.3675\end{aligned}$$

3.

$$\begin{aligned}x + \sin(y) y' &= 2 \\M(x) &= x - 2 \quad , \quad N(y) = \sin(y) \\\cos(y) &= \frac{1}{2}x^2 - 2x - C \\y(x) &= \cos^{-1}\left(\frac{1}{2}x^2 - 2x - C\right)\end{aligned}$$

4.

$$\begin{aligned}xy' - y &= x \quad , \quad y(1) = 0 \\y' - \frac{1}{x}y &= 1 \quad \Rightarrow \quad p(x) = -\frac{1}{x} \quad , \quad g(x) = 1 \\\mu(x) &= \frac{1}{x} \\y(x) &= x \int \frac{1}{x} (1) dx + Cx = x \ln|x| + Cx\end{aligned}$$

5.

$$y + (x + 1) \frac{dy}{dx} = x \quad , \quad y(0) = 1$$

$$\begin{aligned}\frac{\partial M}{\partial y} &= 1 = \frac{\partial N}{\partial x} \\\Phi(x, y) &= xy - \frac{1}{2}x^2 + y \\C &= 1 \\y &= \frac{1 + \frac{1}{2}x^2}{x + 1}\end{aligned}$$

6.

$$(*) \quad x^2 \frac{dy}{dx} = xy - y^2$$

Dividing both sides of the ODE by  $x^2$  yields

$$\begin{aligned} \frac{dy}{dx} &= \frac{y}{x} - \left(\frac{y}{x}\right)^2 \\ x \frac{dz}{dx} + z &= z - z^2 \\ \ln|x| - \frac{1}{z} &= C \\ y(x) &= \frac{x}{\ln|x| - C} \end{aligned}$$