

1.

- (a) $\frac{d^2x}{dt^2} + tx = \sin(x)$: 2nd order, linear, ODE
 (b) $\frac{\partial^3\psi}{\partial^3x} - x^2\frac{\partial\psi}{\partial y} = \psi^2$: 3rd order, nonlinear, PDE
 (c) $\frac{d^3x}{dt^3} + t^2\frac{dx}{dt} + x = 0$: 3rd order, nonlinear, ODE
 (d) $x^2y' + y = e^x \sin(x)$: 1st order, linear, ODE
 (e) $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial\phi}{\partial y} = (x+y)^2$: 2nd order, linear, PDE

2.

$$\begin{aligned} F(x, y) &= xy \\ y_1 &= 2.2 \\ y_2 &= 2.442 \\ y_3 &= 2.735 \end{aligned}$$

3. $3x^2 - e^{2y}y' = x$

$$\begin{aligned} M(x) &= 3x^2 - x, \quad N(y) = -e^{2y} \\ x^3 - \frac{1}{2}x^2 - \frac{1}{2}e^{2y} &= C \\ y(x) &= \frac{1}{2} \ln |2x^3 - x^2 - 2C| \end{aligned}$$

4. $xy' - 2y = x^2, \quad y(1) = 2$

$$\begin{aligned} y' - \frac{2}{x}y &= x \Rightarrow p(x) = -\frac{2}{x}, \quad g(x) = x \\ \mu(x) &= \exp \left[\int -\frac{2}{x} dx \right] = x^{-2} \\ y(x) &= x^2 \int (x^{-2})(x) dx + Cx^2 = x^2 \ln|x| + Cx^2 \end{aligned}$$

5. $3x^2 + (2y - 2x) \frac{dy}{dx} = 2y, \quad y(1) = 2$

$$\begin{aligned} M(x, y) &= 3x^2 - 2y, \quad N(x, y) = 2y - 2x \Rightarrow \frac{\partial M}{\partial y} = -2 = \frac{\partial N}{\partial x} \\ \Phi(x, y) &= \int (3x^2 - 2y) \partial x + c_1(y) = x^3 - 2yx + c_1(y) \\ \Phi(x, y) &= \int (2y - 2x) \partial y + c_2(x) = y^2 - 2xy + c_2(x) \\ \Phi(x, y) &= x^3 - 2xy + y^2 = C \\ y(x) &= x + \sqrt{x^2 - x^3 + 1} \end{aligned}$$

6. $x \frac{dy}{dx} = x + y$

$$\frac{dy}{dx} = 1 + \left(\frac{y}{x}\right)$$

$$x \frac{dz}{dx} + z = z + 1$$

$$-\frac{1}{x} + \frac{dz}{dx} = 0$$

$$-\ln|x| + z = C$$

$$y = xz = x \ln|x| + Cx$$