Math 2233.21570 FIRST EXAM February 23, 2021

Name:

1. (15 pts) Classify the following differential equations: determine their order, if they are linear or non-linear, and if they are ordinary differential equations or partial differential equations.

- (a) $\frac{dx^2}{dt^2} + tx = \sin\left(x\right)$
- (b) $\frac{\partial^3 \psi}{\partial^3 x} x^2 \frac{\partial \psi}{\partial y} = \psi^2$
- (c) $\frac{d^3x}{dt^3} + t^2 \frac{dx}{dt} + x = 0$

(d)
$$x^2y' + y = e^x \sin(x)$$

(e) $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \phi}{\partial y} = (x+y)^2$

2. (15 pts) Consider the following first order ODE: y' = xy and suppose y(x) is the solution satisfying y(1) = 2. Use the numerical (Euler) method with n = 3 and $\Delta x = 0.1$ to estimate y(1.3).

3. (15 pts) Find an explicit solution of the following (separable) differential equation. $3x^2 - e^{2y}y' = x$

4. (15 pts) Solve the following initial value problem

 $xy' - 2y = x^2$, y(1) = 2

5. (20 pts) Consider the following initial value problem

$$3x^{2} + (2y - 2x)\frac{dy}{dx} = 2y$$
$$y(1) = 2$$

Show that the differential equation is exact and then find the explicit solution satisfying the initial condition.

6. (20 pts) Find the general solution of

$$x\frac{dy}{dx} = x + y$$

(Hint: recast the differential equation into the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ and then use the change of variable $z\left(x\right) = \frac{y(x)}{x}$.)