Math 2233.21570
Final Exam
2:00pm – 4:00pm
May 4, 2021

Name:________________________________

1. (15 pts) Solve the following initial value problem

$$xy' - 3y = x^2 \qquad , \qquad y(1) = 3 \quad .$$

2. (15 pts) Show that the following equation is exact and then find the explicit solution.

$$(3x+1)\,\frac{dy}{dx} = 2x - 3y$$

3. (15 pts) Given that $y_1(x) = e^{2x}$ is one solution of

(*) $$y'' - 4y' + 4y = 0 \quad ,$$

use Reduction of Order (explicitly) to find the general solution of (*).

4. (20 pts) Solve the following initial value problem

$$y'' - 2y' - 3y = e^{2x} \quad , \quad y(1) = 2 \quad , \quad y'(1) = 1$$

(Hint: the corresponding homogeneous equation is a constant coefficients type ODE)

5. (15 pts) Find the function $f(x)$ whose Laplace transform is $F(s) = \dfrac{s+1}{(s+2)(s+3)}$.

6. (15 pts) Solve the following initial value problems using the Laplace transform method.

$$\begin{aligned} y'' + 3y' + 2y &= 0 \\ y(0) &= 2 \\ y'(0) &= -4 \end{aligned}$$

7. (15 pts) Reduce the following expression to a single power series:

$$x\sum_{n=0}^{\infty} a_n\,(x+2)^n + \sum_{n=0}^{\infty} a_n\,(x+2)^n$$

8. (15 pts) Find the recursion relations for a power series solution about $x_o = 0$ for the following differential equation.

$$y'' - (x+1)\,y' - y = 0$$

9. (10 pts) **Given** that the recursion relations for $y'' - 2xy' - 2y = 0$ about $x_o = 1$ are

$$a_{n+2} = \frac{2a_n + 2a_{n+1}}{n+2} \qquad , \qquad n = 0, 1, 2, 3, \ldots$$

Write down the first 4 terms of the power series solution satisfying $y(1) = 1$, $y'(1) = 2$. (I.e., find the solution up to order $(x-1)^3$.)

10. Consider the following differential equation:

$$\left(1 - x^2\right) y'' + xy' + \frac{1}{(x-2)} y = 0$$

(a) (5 pts) Identify the singular points of this differential equation.

(b) (10 pts) For what range of x is a power series solution of the form $\sum_{n=0}^{\infty} a_n (x+2)^n$ guaranteed to converge?

Table of Laplace Transforms

$$\mathcal{L}\left[\alpha f(t) + \beta g(t)\right] = \alpha\mathcal{L}\left[f(t)\right] + \beta\mathcal{L}\left[g(t)\right] \qquad \text{if } \alpha, \beta \text{ are constants}$$

$$\mathcal{L}\left[\frac{df}{dt}\right] = s\mathcal{L}\left[f\right] + f\left(0\right)$$

$$\mathcal{L}\left[\frac{d^2 f}{dx^2}\right] = s^2\mathcal{L}\left[f\right] + sf\left(0\right) + \frac{df}{dt}\left(0\right)$$

$$\mathcal{L}\left[t^n\right] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\left[e^{at}\right] = \frac{1}{s-a}$$

$$\mathcal{L}\left[\sin(at)\right] = \frac{a}{s^2+a^2}$$

$$\mathcal{L}\left[\cos(at)\right] = \frac{s}{s^2+a^2}$$

$$\mathcal{L}\left[\sinh\left(at\right)\right] = \frac{a}{s^2-a^2}$$

$$\mathcal{L}\left[\cosh\left(at\right)\right] = \frac{s}{s^2-a^2}$$

$$\mathcal{L}\left[e^{at}\sin\left(bt\right)\right] = \frac{b}{\left(s-a\right)^2+b^2}$$

$$\mathcal{L}\left[e^{at}\cos\left(bt\right)\right] = \frac{s-a}{\left(s-a\right)^2+b^2}$$

$$\mathcal{L}\left[t^n e^{at}\right] = \frac{n!}{\left(s-a\right)^{n+1}}$$

$$\mathcal{L}\left[e^{at}\sinh\left(bt\right)\right] = \frac{b}{\left(s-a\right)^2-b^2}$$

$$\mathcal{L}\left[e^{at}\cosh\left(bt\right)\right] = \frac{s-a}{\left(s-a\right)^2-b^2}$$

$$\mathcal{L}\left[u\left(t-a\right)f\left(t\right)\right] = e^{-at}\mathcal{L}\left[f\left(t+a\right)\right]$$

$$\mathcal{L}^{-1}\left[e^{-as}\mathcal{L}\left[f\right]\right] = u\left(t-a\right)f\left(t-a\right)$$