Math 2233 - Lecture 1

Agenda:

1. Syllabus and Logistics for the Course
2. How to Succeed in Math 2233
3. Introduction to the Subject Matter of Math 2233
    (a) Differential Equations in the Physical Sciences
    (b) Types of Differential Equations
    (c) Solutions of Differential Equations and Boundary Conditions
    (d) Outline of Course

Syllabus and Logistics

Syllabus and Logistics

- Instructor: Dr. Birne Binegar
  `birne.binegar@okstate.edu`
  425 MSCS
  405-744-5793 (direct to my cellphone)
- Canvas: Canvas is the learning platform used for online courses at OSU. To access Canvas, students should visit https://canvas.okstate.edu/ in a web browser. All enrolled students should automatically have access to the Math 2233 course materials that will appear on their Canvas "Courses" page.
- Lectures: Lectures will be conducted online via Zoom.
  - Section 21569: T,Th at 10:30am CDT:
    Zoom Meeting ID: 973 9594 5989 ,
    Passcode: 429180
  - Section 21570: T,Th at 1:30pm CDT:
    Zoom Meeting ID: 967 1845 6559 ,
    Passcode: 349478

Syllabus and Logistics: Additional Resources

- Lecture Recordings:
  - The lectures for each section will be recorded and posted on Canvas.
  - However, students are expected to participate in the lectures as they occur. In order to participate in the lectures, a computer (or smartphone) with a web browser and a microphone, will be needed. A webcam would also be useful for in-class discussions.
- Office Hours:
  Office hours will be at 2:00pm CDT, on Mondays and Wednesdays and will conducted via Zoom.
  - Zoom Meeting ID: 942 5647 4563
  - Zoom Meeting Passcode: 429180
- MLSC : Online tutoring will be available. More info to follow.

Syllabus and Logistics: Exams

- Exams :
  - This course will have 2 midterm exams and a final exam.
  - Each of these exams will conducted via Canvas.
  - The midterm exams will be held during regular class times and their dates will be announced well in advance.
  - The final exams will be held on Tuesday, May 4 at the following times:
    - The final exam for Section 21569 (10:30 T,Th) will be held Tuesday, May 4, 10:00 am – 11:50 am
    - The final exam for Section 21570 (1:30 T,Th) will be held Tuesday, May 4, 2:00 pm - 3:50 pm

Syllabus and Logistics: Exams Logistics and Homework

- The exams will be conducted as follows.
  - At the scheduled day and time, the exam questions will be posted on Canvas.
  - Students will then have 90 minutes (for midterms) or 2 hours (for final exam) to download the exam questions, complete the exam, and then upload their answers back to Canvas.
  - Students will need to scan (or digitally photograph) their answers, convert the page images to a single pdf file, and then upload the pdf file to Canvas.
  - Most smartphones have apps for doing this (e.g. CamScanner for Android, or PDF Expert on iPhones or iPads)
- Homework:
  Homework will be assigned weekly and the assignments will problem sets on MyLab Math (the online homework submission package that comes with the textbook).

Syllabus and Logistics: Final Grades

- Grading: Final grades will be based on the scores attained in their homework assignments, midterm exams, and final exam.

1st Midterm Examination	100 possible pts.
2nd Midterm Examination	100 possible pts.
Final Examination	150 possible pts.
Homework	25 possible pts.
	375 possible pts.

Letter grades will be assigned as follows:

A:	337	-	375 pts.
B:	300	-	336 pts.
C:	262	-	299 pts.
D:	225	-	2261 pts.
F:	0	-	224 pts.

Questions about how the course will run?

How to Succeed in Math 2233

- Download and print the lecture slides before lecture and then add supplemental notes directly to your copy during lecture.
- Participate in class and feel free to interupt when things are not clear
- I will try to promote an active learning environment in class - and to make learning efficient in class.
- Keep up with the homework assignments. The problems there are vital preparation for the exams.
- Take advantage of the free online tutoring available from the MLSC.

Why Are We Here? : Differential Equations and the Physical Sciences

- Functions are used to model how one physical quantity depends on another.
- Physical Laws are generally stated as differential equations that govern such functions.
  E.g., $\mathbf{F} = m\mathbf{a} \equiv m\frac{d^2x}{dt^2}$
- Specific experimental situations correspond to placing "initial conditions" on solutions to differential equations.

Example: A Falling Object

A function $z(t)$ is used to describe the vertical position of an object as a function of the time t.
In the presence of gravity (near the earth's surface) one has

$$\text{Newton's 2nd Law of Motion : } \mathbf{F} = m\mathbf{a}$$

$$\implies \quad -mg = m\frac{d^2 z}{dt^2}$$

General Solution of the Differential Equation:

$$z(t) = c_1 + c_2 t - \frac{1}{2}gt^2$$

Solution corresponding to specific experiment:

$$\begin{aligned} z(0) &= c_1 & \implies & \quad c_1 \text{ is the initial height} \\ \frac{dz}{dt}(0) &= c_2 & \implies & \quad c_2 \text{ is the initial velocity} \end{aligned}$$

Differential Equations and Physical Applications: Summary

$$\text{Physical Law} \quad \Longleftrightarrow \quad \text{Differential Equation}$$

$$\infty\text{-Many Experimental Situations} \quad \Longleftrightarrow \quad \infty\text{-Many Solutions}$$

$$\text{particular experiment} \quad \Longleftrightarrow \quad \text{solution satisfying fixed initial conditions}$$

Solutions of Differential Equations

Definition

A **solution** of a differential equation is a *function* for which the differential equation is automatically satisfied upon substitution into the equation.

Example: Consider

$$\frac{dy}{dx} = 3y$$

Every function of the form

$$y(x) = Aexp(3x) \quad , \qquad \text{where } A \text{ is any constant number}$$

is a solution. For when $y(x) = Ae^{3x}$

$$\begin{aligned} \frac{d}{dx}(y(x)) &= \frac{d}{dx}\left(Ae^{3x}\right) = A\left(3e^{3x}\right) \\ &= 3\left(Ae^{3x}\right) = 3y(x) \end{aligned}$$

$$\Longrightarrow \quad \frac{dy}{dx}(x) = 3y(x) \quad \text{for each } x \in \mathbb{R}$$

Remarks

- Note that satisfaction of the differential equation implies an infinite number of numerical conditions on the solution. (since the solution function must satisfy the DE for all values of x in its domain)
- Note that there are also an infinite number of solutions (since there are ∞ many choices for the constant A)
- It is most apt to talk about the **dimension** of the solution space rather than the number of solutions.
- Specifying a numerical value for the constant A $\longrightarrow$ choosing a "**particular solution**" of the DE
- Leaving A as an unspecified parameter allows us to talk about **all** solutions of the DE simultaneously. This is what we shall mean by the "**general solution**" to a differential equation.

Solving Differential Equations by Formulating an Ansatz

It turns out that there exists no single method for solving differential equations. However, the following strategy is quite successful:

- Use the *form* of a differential equation to make an educated guess as to the functional form of a solution. In formulating such a guess, one introduces certain constants to be used as adjustable parameters. Such a guess is referred to as **ansatz** for the solution (and this step is called **making an ansatz for the solution**).
- Plug the ansatz into both sides of the differential equation and try to adjust the parameters in such a way that the ansatz actually becomes a solution ("**fulfilling the ansatz**")
- Make a theoretical argument as to why the solutions found in this way are **all** of the solutions of the differential equation.

Example: Making an Ansatz

Consider again

$$\frac{dy}{dx} = 3y \tag{1}$$

To satisfy this differential equation a function $y(x)$ must have the property that its derivative is just a constant times itself. of the form

$$y(x) = Ae^{\lambda x} \quad A, \lambda \text{ constants to be determined}$$

Any function of this form will satisfy

$$\frac{dy}{dx} = \frac{d}{dx}(Ae^{\lambda x}) = \lambda Ae^{\lambda x} = \lambda y(x)$$

And so by setting $\lambda = 3$, we can make $y(x) = Ae^{\lambda x}$ satisfy (1). Thus, we have found a set of solutions of the form

$$y(x) = Ae^{3x}$$

(We still need a theoretical argument, however, to conclude that we have found all the solutions.)

Ansatzs and Problem Solving

As this course proceeds, this ansatz methodology will be used over and over again.

Indeed, this idea of formulating ansatzs in order to solve differential equations may be the most important idea of the course.

It is the essence of how one leverages mathematical models to solve physical problems.

Types of Differential Equations

Since the first step of our method for solving DEs will be

$$\text{form of the differential equation} \quad \Longrightarrow \quad \text{ansatz for the solutions}$$

our solution strategies will depend critically on the form of the differential equation.

We therefore begin the course with a rough accounting of the different types of differential equations.

Classifying Differential Equations: Ordinary vs. Partial Differential Equations

A differential equation, of course, involves the derivatives of a function. But there are two kinds of derivatives:

- Ordinary Derivatives: for functions of a single variable
  Notations: $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$, etc
- Partial Derivatives: for functions of more than one variable
  Notations: $\frac{\partial\phi}{\partial x} = \partial_x\phi = \phi_x$, $\frac{\partial^2\phi}{\partial x\partial y} = \partial_x\partial_y\phi = \phi_{xy}$, etc.

In either case, the function being differentiated plays the role of the **unknown** in the differential equation, while the variable(s) that's doing the differentiation plays the role of the **underlying variable(s)**.

At any rate, we can distinguish between differential equations by how many underlying variables are involved.

- Only one underlying variable $\Longrightarrow$ an **ordinary differential equation** or **ODE**
- More than one underlying variable $\Longrightarrow$ a **partial differential equation** or **PDE**

Classifying Differential Equations: The Order of a Differential Equation

The **order** of a differential equation is simply the degree of the highest derivative that occurs in the differential equation.

Examples

$$\begin{aligned} y'' + 2xy' + y &= 0 \quad \Rightarrow \quad 2^{nd} \text{ order ODE} \\ \frac{\partial^3 \phi}{\partial x \partial y \partial z} + \left(\frac{\partial \phi}{\partial x}\right)^2 &= xy^2 z \quad \Rightarrow \quad 3^{rd} \text{ order PDE} \end{aligned}$$

Classifying Differential Equations: Linear vs. Nonlinear Differential Equations

A third general property of a differential equation is whether it is a **linear** or a **nonlinear** differential equation.

This distinction is a bit subtle at first. However, once you grasp the form of a linear differential equation, it is a really easy distinction to make.

Let me start by talking about *linear functions*.

A function f of a single variable x is **linear** if it has the following functional form

$$f(x) = Ax + B \quad , \quad A, B \; constants \qquad (*)$$

Thus, f(x) = 3x +2 is a linear function, since $f(x)$ is of the form (*) with $A = 3$ and $B = 2$.

While $g(x) = cos(x)$ or $h(x) = 3x^2$ are *nonlinear* functions of x, since neither of these functions can be cast in the form $Ax + B$

Linear Functions, Cont'd

An alternative characterization of a linear function is that it is satisfies the differential equation

$$\frac{d^2}{dx^2} f(x) = 0$$

This property generalizes to functions of several variables as follows :

Definition

Let ϕ be a function of n variables $x_1, \ldots, x_n$. We say that ϕ is **simultaneously linear** with respect to say $x_1, x_2, \ldots, x_k$ if

$$\frac{\partial^2 \phi}{\partial x_i \partial x_j} = 0 \quad , \quad \text{for all } i, j \in \{1, \ldots, k\}$$

Linear Functions, Cont'd

Thus, a function $\phi(x, y, z)$ of 3 variables is simultaneously linear with respect to y, z if it has the form

$$\phi\left(x,y,z\right)=A\left(x\right)y+B\left(x\right)z+C\left(x\right)$$

This is because the linearity condition requires all the double derivatives with respect to y and/or z must vanish identically;

and this can only happen if each term of $\phi(x, y, z)$ has **at most** one factor of y or z and no terms with both y and z as factors.

Linear Differential Equations

Definition

A differential equation

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots\right) = 0$$

is **linear** if the defining function F is simultaneously linear with respect to $y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots$.

Note that this definition **ignores how F depends on the underlying variable** x. It is only concerned with how F depends on the unknown function y and its derivatives.

Linear vs Nonlinear ODEs; Examples

Consider

$$x^2 y' + xy = \cos(x)$$

Rewriting this as

$$x^2 y' + y - \cos(x) = 0$$

we see that the function $F(x, y, y')$ that prescribes this differential equation is

$$F(x, y, w) = x^2 w + xy - \cos(x)$$

(I've relabeled y' as w to stress its interpretation as a variable of the function F that defines the DE) and it satisfies

$$\frac{\partial^2 F}{\partial y^2} = 0 \quad , \quad \frac{\partial^2 F}{\partial y \partial w} = 0 \quad , \quad \frac{\partial^2 F}{\partial w^2} = 0$$

and so $F(x, y, w)$ is simultaneously linear w.r.t. y and w. And so $F(x, y, y')$ is simultaneously linear w.r.t. y and its derivative(s). Thus, the original differential equation is a linear ODE.

Upshot: Linear ODEs

Just as a linear function $f(x) = Ax + B$ is about the simplest kind of function (aside from constant functions), linear differential equations form a comparatively simple kind of differential equation.

It is a remarkable and extremely lucky circumstance that most of the fundamental differential equations governing physical phenomena are linear differential equations.

Because of their simplicity and pervasiveness - this course will concentrate on linear ODEs.

Rough Outline of Course

1. 1st Order ODEs
   - Linear 1st order ODEs (complete solution in 2 lectures)
   - Nonlinear 1st order ODEs (even several weeks of lecture only special cases can be handled)
2. 2nd Order ODEs
   - Last 2/3 of course is dedicated to solving 2nd order linear ODEs