Math 2233 - Lecture 1

Agenda:

1. Syllabus and Logistics for the Course
2. How to Succeed in Math 2233
3. Introduction to the Subject Matter of Math 2233
   (a) Differential Equations in the Physical Sciences
   (b) Types of Differential Equations
   (c) Solutions of Differential Equations and Boundary Conditions
   (d) Outline of Course

Syllabus and Logistics

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- Instructor: Dr. Birne Binegar
  `birne.binegar@okstate.edu`
  425 MSCS
  405-744-5793 (direct to my cellphone)

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- Lectures: Lectures will be conducted online via Zoom.
  - Section 21569: T,Th at 10:30am CDT:
    Zoom Meeting ID: 973 9594 5989 ,
    Passcode: 429180
  - Section 21570: T,Th at 1:30pm CDT:
    Zoom Meeting ID: 967 1845 6559 ,
    Passcode: 349478

Syllabus and Logistics: Additional Resources

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- MLSC : Online tutoring will be available. More info to follow.

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    - The final exam for Section 21569 (10:30 T,Th) will be held Tuesday, May 4, 10:00 am – 11:50 am
    - The final exam for Section 21570 (1:30 T,Th) will be held Tuesday, May 4, 2:00 pm - 3:50 pm

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  Homework will be assigned weekly and the assignments will problem sets on MyLab Math (the online homework submission package that comes with the textbook).

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A:	337	-	375 pts.
B:	300	-	336 pts.
C:	262	-	299 pts.
D:	225	-	2261 pts.
F:	0	-	224 pts.

Questions about how the course will run?

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- Take advantage of the free online tutoring available from the MLSC.

Why Are We Here? : Differential Equations and the Physical Sciences

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  E.g., $\mathbf{F} = m\mathbf{a} \equiv m\frac{d^2x}{dt^2}$
- Specific experimental situations correspond to placing "initial conditions" on solutions to differential equations.

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General Solution of the Differential Equation:

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Solution corresponding to specific experiment:

$$\begin{aligned} z(0) &= c_1 \quad \Longrightarrow \quad c_1 \text{ is the initial height} \\ \frac{dz}{dt}(0) &= c_2 \quad \Longrightarrow \quad c_2 \text{ is the initial velocity} \end{aligned}$$

Differential Equations and Physical Applications: Summary

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particular experiment $\iff$ solution satisfying fixed initial conditions

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- It is most apt to talk about the **dimension** of the solution space rather than the number of solutions.
- Specifying a numerical value for the constant $A \quad \longrightarrow$ choosing a "**particular solution**" of the DE
- Leaving A as an unspecified parameter allows us to talk about **all** solutions of the DE simultaneously. This is what we shall mean by the "**general solution**" to a differential equation.

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- Plug the ansatz into both sides of the differential equation and try to adjust the parameters in such a way that the ansatz actually becomes a solution ("**fulfilling the ansatz**")
- Make a theoretical argument as to why the solutions found in this way are **all** of the solutions of the differential equation.

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(We still need a theoretical argument, however, to conclude that we have found all the solutions.)

Ansatzs and Problem Solving

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It is the essence of how one leverages mathematical models to solve physical problems.

Types of Differential Equations

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We therefore begin the course with a rough accounting of the different types of differential equations.

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  Notations: $\frac{\partial\phi}{\partial x} = \partial_x\phi = \phi_x$, $\frac{\partial^2\phi}{\partial x\partial y} = \partial_x\partial_y\phi = \phi_{xy}$, etc.

In either case, the function being differentiated plays the role of the **unknown** in the differential equation, while the variable(s) that's doing the differentiation plays the role of the **underlying variable(s)**.

At any rate, we can distinguish between differential equations by how many underlying variables are involved.

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Examples

$$\begin{aligned} y'' + 2xy' + y &= 0 \quad \Rightarrow \quad 2^{nd} \text{ order ODE} \\ \frac{\partial^3 \phi}{\partial x \partial y \partial z} + \left(\frac{\partial \phi}{\partial x}\right)^2 &= xy^2 z \quad \Rightarrow \quad 3^{rd} \text{ order PDE} \end{aligned}$$

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While $g(x) = cos(x)$ or $h(x) = 3x^2$ are *nonlinear* functions of x, since neither of these functions can be cast in the form $Ax + B$

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and this can only happen if each term of $\phi(x, y, z)$ has **at most** one factor of y or z and no terms with both y and z as factors.

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A differential equation

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and so $F(x, y, w)$ is simultaneously linear w.r.t. y and w. And so $F(x, y, y')$ is simultaneously linear w.r.t. y and its derivative(s). Thus, the original differential equation is a linear ODE.

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It is a remarkable and extremely lucky circumstance that most of the fundamental differential equations governing physical phenomena are linear differential equations.

Because of their simplicity and pervasiveness - this course will concentrate on linear ODEs.

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   - Last 2/3 of course is dedicated to solving 2nd order linear ODEs