Math 2233 - Lecture 8: Review Session for 1st Exam

Agenda:

1. Procedure for taking 1st exam.
2. Overview of topics to appear on 1st Exam
3. Practice Exam

Procedure for Taking the 1st Exam

The 1st exam will be conducted as follows.

1. On Tuesday, Feb. 23 at regular class meeting time (10:30am CST for Math2233.51569, 1:30pm CST for Math2233.21570), I will post the exam on Canvas (at the top of the Home page for the course)
2. Students will then have 90 minutes to
   - Download the exam questions.
   - Complete the exam.
   - Scan or digitally photograph their answers and convert the page images to a single PDF file.
   - Submit their test papers by uploading the PDF using the **Assignments** section of Canvas.

Remarks:

- Students are encouraged to try carrying out a practice submission before the actual exam date. I have posted a Practice Submission assignment to the Assignment sections of Canvas for this purpose.
- To carry out the third step of the submission procedure, students will need to scan (or digitally photograph) their exams, convert the page images to a single pdf file, and then upload the pdf file to Canvas.
  Most smartphones have apps for doing this (e.g. CamScanner for Android, or PDF Expert on iPhones or iPads)

Topics to be Covered on 1st Exam

1. Classification of Differential Equations
   - ODE vs. PDE
   - The Order of a Differential Equation
   - Linear vs. Nonlinear Differential Equations
2. The Numerical (Euler) Method
3. Separable Equations
4. Linear 1st Order ODEs
5. Exact Equations
6. Change of Variables

The Numerical (Euler) Method

Given a 1st order ODE with initial conditions

$$\frac{dy}{dx} = F(x,y) \qquad , \qquad y(x_0) = y_0 \tag{1}$$

one can construct a table of approximate values

x	**y**
x_0	y_0
$x_1 = x_1 + \Delta x$	$y_1 = y_0 + F(x_0, y_0)\,\Delta x$
$x_2 = x_1 + \Delta x$	$y_2 = y_1 + F(x_1, y_1)\,\Delta x$
$x_3 = x_2 + \Delta x$	$y_3 = y_2 + F(x_2, y_2)\,\Delta x$
$\vdots$	$\vdots$

(with each $y_i \approx y(x_i)$).

The Numerical Method in More Detail

- Choose a small step size Δx
- The first line of the table is just values of x and y prescribed by the initial condition $y(x_0) = y_0$
- The rest of the entries are determined iteratively by the formulas

$$\begin{aligned} x_{i+1} &= x_i + \Delta x \\ y_{i+1} &= y_i + F(x_i, y_i)\,\Delta x \end{aligned}$$

Separable Equations

Standard Form:

$$M(x) + N(y)\frac{dy}{dx} = 0 \tag{2}$$

Separable ODEs are derivable from algebraic equations of the form

$$H_1(x) + H_2(y) = C \tag{3}$$

Solution Method:

- Transform ODE into form (2) and identify the functions $M(x)$ and $N(y)$ correctly.
- Calculate

  $$H_1(x) = \int M(x)\,dx \quad , \quad H_2(y) = \int N(y)\,dy$$

  and plug your results into the equation (3)
  This will yield the **implicit solution** of (3)
- If possible, solve (algebraically) the implicit solution to determine y as a function of x and C.

Linear 1st Order ODEs

Standard Form:

$$\frac{dy}{dx} + p(x)y = g(x) \tag{4}$$

Solution Method:

- Transform linear ODE into the standard form (4) to correctly identify the coefficient functions $p(x)$ and $g(x)$
- Compute the **integrating factor** $\mu(x)$

$$\mu(x) = \exp\left[\int p(x)\,dx\right] \tag{5}$$

- Compute the general solution of (4) as

$$y(x) = \frac{1}{\mu(x)} \int \mu(x)g(x)\,dx \quad + \quad \frac{C}{\mu(x)} \tag{6}$$

Exact Equations

Standard Form:

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0 \quad \text{with} \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \tag{7}$$

Exact Equations are ODEs that are derivable from algebraic equations of the form

$$\Phi\,(x,y) = C \tag{8}$$

Method:

- Verify that the equation is exact (i.e., $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$)
- Compute

$$\begin{aligned} \Phi_1(x,y) &= \int M(x,y)\,\partial x \; + \; c_1(y) \\ \Phi_2(x,y) &= \int N(x,y)\,\partial y \; + \; c_2(x) \end{aligned}$$

- Adjust the arbitrary functions $c_1(y)$ and $c_2(x)$ so that $\Phi_1(x,y) = \Phi_2(x,y) \equiv \Phi(x,y)$

Exact Equations, Cont'd

- Insert the calculated $\Phi(x, y)$ into

$$\Phi(x, y) = C$$

  to obtain the **implicit solution** of (7)
- If possible, solve the implicit solution for y as a function of x and C.

Change of Variables for ODEs of Homogeneous Type

Standard Form:

$$\frac{dy}{dx} = F\left(\frac{y}{x}\right) \tag{9}$$

Such ODEs can always be coverted to an **Separable ODE** by making a change of variables $z(x) = \frac{y(x)}{x}$.

Solution Method:

1. Introduce auxiliary function $z(x) = \frac{y(x)}{x}$.
2. Re-express y in terms of z and x and compute how to express $\frac{dy}{dx}$ in terms of x, z, and $\frac{dz}{dx}$

$$y = zx \quad \Longrightarrow \quad \frac{dy}{dx} = x\frac{dz}{dx} + z$$

3. Substitute $x\frac{dz}{dx} + z$ for $\frac{dy}{dx}$ on the L.H.S of (9) and z for $\frac{y}{x}$ on the R.H.S. of (9).
4. Solve the resulting Separable ODE for $z(x)$
5. Replace the LHS of your equation for $z(x)$ with $\frac{y}{x}$ and then solve for y. The corresponding function $y(x)$ will be the solution to (9).