Math 3013
FIRST EXAM
12:30 - 1:20pm, September 15, 2021

Name:________________________________

1. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \quad , \quad \mathbf{B} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

Compute the matrix products **AB** and **BA** (if they exist)

2. For each of the following augmented matrices, indicate

- the number of equations and the number of variables in the corresponding linear system
- whether or not the corresponding linear system has a solution
- if the corresponding linear system does have a solution, the number of free variables in the solution.

Hint: note that these augmented matrices are already in row echelon form and you do not have to actually solve the systems.

(a) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$

(b) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 3 & 1 \end{array}\right]$

3. Consider the following linear system

$$\begin{aligned} x_2 + 2x_3 + x_4 &= 1 \\ 2x_1 + x_2 + x_3 + 2x_4 &= -1 \\ x_1 + x_2 + 2x_3 - x_4 &= 3 \end{aligned}$$

Write down the corresponding augmented matrix and row reduce it to row-echelon form.

4. Row reduce the following augmented matrix to **reduced** row-echelon form.

$$[\mathbf{A} \mid \mathbf{b}] = \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 2 & 2 \\ 0 & 0 & 2 & 4 & 4 & 4 \\ 0 & 0 & 0 & 1 & -2 & 2 \end{array}\right]$$

5. Suppose the augmented matrix below is the Reduced Row Echelon Form of an augmented matrix of a linear system. Display the solution of the linear system as a hyperplane (within the space of variables).

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

6. Determine if the matrix below has an inverse and, if so, determine its inverse. (Hint: a single row reduction process will answer both parts of to this problem.)

$$\mathbf{A} = \left[\begin{array}{ccc} 1 & -1 & 1 \\ -2 & 2 & -1 \\ 1 & 0 & 0 \end{array}\right]$$