Math 3013.21403
SECOND EXAM
March 28, 2022

Calculators and/or notes are not permitted for this exam.

Name:______________________________

1. Define, precisely, the following notions (where V, W are to be regarded as general vector spaces). space V).

(a) (5 pts) a **subspace** of V is ...

(b) (5 pts) a **basis** for a vector space V is ...

(c) (5 pts) a **set of linearly independent vectors** in V is ...

(d) (5 pts) a **linear transformation** from a vector space V to vector space W is ...

2. (10 pts) Prove or disprove that the points on the set $S = \{[x, y] \in \mathbb{R}^2 \mid y = x + 1\}$ is a subspace of $\mathbb{R}^2$.

3. (10 pts) Let $W = span\left([1,1,1,1],[1,-1,1,0],[2,0,2,1]\right) \subset \mathbb{R}^4$. Find a basis for W

4. Consider the following matrix: $\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 & 2 \\ 2 & 2 & 0 & 4 \\ 0 & 2 & 0 & 2 \end{bmatrix}$

(a) (10 pts) Row reduce this matrix to reduced row echelon form

(b) (5 pts) Find a basis for the row space of $\mathbf{A}$.

(c) (5 pts) Find a basis for the column space of $\mathbf{A}$.

(d) (5 pts) Find a basis for the null space of $\mathbf{A}$.

(e) (5 pts) What is the rank of $\mathbf{A}$?

5. (10 pts) Let $\mathbf{A}$ be an $n \times m$ matrix. Show that the function $T_{\mathbf{A}} : \mathbb{R}^m \to \mathbb{R}^n : T(\mathbf{x}) = \mathbf{A}\mathbf{x}$ (via matrix multiplication on the right) is a linear transformation.

6. Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2 : T([x_1, x_2, x_3]) = [x_1 - x_3, x_2 + x_3]$.
(a) (10 pts) Find the matrix $\mathbf{A}_T$ corresponding to T:

(b) (5 pts) Find a basis for $range(T) \equiv \{\mathbf{y} \in \mathbb{R}^2 \mid \mathbf{y} = T(\mathbf{x}) \text{ for some } \mathbf{x} \in \mathbb{R}^3\}$.

(c) (5 pts) Find a basis for $ker(T) \equiv \{\mathbf{x} \in \mathbb{R}^4 \mid T(\mathbf{x}) = \mathbf{0}\}$