Math 3013

SECOND EXAM

November 3, 2020

Name:______________________________

1. Consider the vectors $\{[1,2,1,1,0]\ ,\ [0,1,2,1,1]\ ,\ [-1,-1,1,0,1]\ ,\ [-1,0,3,1,2]\} \in \mathbb{R}^4$

(a) (10 pts) Determine if these vectors are linearly independent.

(b) (5 pts) What is the dimension of the subspace generated by these vectors?

2. Write the definitions (as stated in class) of the following notions. (5 pts each)

(a) A **subspace** of $\mathbb{R}^n$.

(b) A **basis** for a subspace of $\mathbb{R}^n$.

(c) A set of **linearly independent vectors**

(d) A **linear transformation**

3. Given that the matrix $\mathbf{A} = \begin{bmatrix} 2 & 4 & 0 & 2 \\ -1 & -2 & 1 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a) (5 pts) Find a basis for the row space of $\mathbf{A}$.

(b) (5 pts) Find a basis for the column space of $\mathbf{A}$.

(c) (5 pts) Find a basis for the null space of $\mathbf{A}$.

(d) (5 pts) What is the rank of $\mathbf{A}$?

4. Consider the following linear transformation:

$$T : \mathbb{R}^3 \to \mathbb{R}^3 : T\left([x_1\ ,\ x_2\ ,\ x_3\right) = [x_1 + 2x_2 + x_3\ ,\ x_1 + x_2\ ,\ x_1 - x_3].$$

(a) (10 pts) Find a matrix that represents T.

(b) (5 pts) Find a basis for the range of T.

(c) (5 pts) Find a basis for the kernel of T.

5. (15 pts) Use a cofactor expansion to compute the determinant of $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 0 & 0 & 2 \end{bmatrix}$

6. (15 pts) Use the row reduction method to determine the determinant of $\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 2 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$