Math 3013 MAKEUP SECOND EXAM March 28, 2022

Calculators and/or notes are not permitted for this exam.

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1. Define, precisely, the following notions (where V, W are to be regarded as general vector spaces). space V).

(a) (5 pts) a subspace of V is ...

- (b) (5 pts) a **basis** for a vector space V is ...
- (c) (5 pts) a set of linearly independent vectors in V is ...
- (d) (5 pts) a linear transformation from a vector space V to vector space W is ...

2. (10 pts) Prove or disprove that the points on the set $S = \{[x, y] \in \mathbb{R}^2 \mid y = x + 1\}$ is a subspace of \mathbb{R}^2 .

3. (10 pts) Let $W = span([1, 1, 1, 1], [1, -1, 1, 0], [2, 0, 2, 1]) \subset \mathbb{R}^4$. Find a basis for W

4. Consider the following matrix: $\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 & 2 \\ 2 & 2 & 0 & 4 \\ 0 & 2 & 0 & 2 \end{bmatrix}$ (a) (10 pts) Row reduce this matrix to reduced row echelon form

- (b) (5 pts) Find a basis for the row space of **A**.
- (c) (5 pts) Find a basis for the column space of **A**.
- (d) (5 pts) Find a basis for the null space of **A**.

(e) (5 pts) What is the rank of \mathbf{A} ?

5. (10 pts) Let **A** be an $n \times m$ matrix. Show that the function $T_{\mathbf{A}} : \mathbb{R}^m \to \mathbb{R}^n : T(\mathbf{x}) = \mathbf{A}\mathbf{x}$ (via matrix multiplication on the right) is a linear transformation.

6. Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2 : T([x_1, x_2, x_3]) = [x_1 - x_3, x_2 + x_3].$ (a) (10 pts) Find the matrix \mathbf{A}_T corresponding to T:

(b) (5 pts) Find a basis for $range(T) \equiv \{ \mathbf{y} \in \mathbb{R}^2 \mid \mathbf{y} = T(\mathbf{x}) \text{ for some } \mathbf{x} \in \mathbb{R}^3 \}.$

(c) (5 pts) Find a basis for $ker(T) \equiv {\mathbf{x} \in \mathbb{R}^3 | T(\mathbf{x}) = \mathbf{0}}$