Math 3013.62663
SECOND EXAM
October 22, 2021

Name:______________________________

1. Complete the following mathematical definitions

(a) (5 pts) A **subspace** of a vector space V is ...

(b) (5 pts) A **basis** for a subspace W is ...

(c) (5 pts) A set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is **linearly independent** if ...

(d) (5 pts) A function $T : \mathbb{R}^m \to \mathbb{R}^n$ is a **linear transformation** if ...

2. Consider the vectors $\{[0, 1, -1, 2], [1, 1, 1, 1], [2, 1, 3, 0]\} \in \mathbb{R}^4$

(a) (5 pts) Determine if these vectors are linearly independent.

(b) (5 pts) What is the dimension of the subspace generated by these vectors?

3. Given that the following matrix: $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 2 & 0 & 2 & 5 \\ 1 & 0 & 1 & 3 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a) (5 pts) Find a basis for the row space of $\mathbf{A}$.

(b) (5 pts) Find a basis for the column space of $\mathbf{A}$.

(c) (5 pts) Find a basis for the null space of $\mathbf{A}$.

(d) (5 pts) What is the rank of $\mathbf{A}$?

4. Consider the following linear transformation:
$T : \mathbb{R}^3 \to \mathbb{R}^3 : T([x_1, x_2, x_3]) = [x_1 + x_2, x_1 - x_3, x_1 + 2x_2 + x_3]$.
(a) (5 pts) Find the matrix $\mathbf{A}_T$ such that $\mathbf{A}_T \mathbf{x} = T(\mathbf{x})$.

(b) (5 pts) Find a basis for the range of T.

(c) (5 pts) Find a basis for the kernel of T.

5. Consider the matrix $\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 3 \\ 0 & 1 & -3 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 \end{bmatrix}$
(a) 7 pts) Compute $\det(\mathbf{A})$ via a cofactor expansion along the third row.

(b) (8 pts) Compute $\det(\mathbf{A})$ by row reducing $\mathbf{A}$ to an upper triangular matrix.

6. (10 pts) Use Cramer's Rule to solve

$$\begin{aligned} x_1 + 2x_2 &= 3 \\ -x_1 + x_2 &= 3 \end{aligned}$$

7. (10 pts) Find all the cofactors of $\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$ and then use these cofactors to compute $\mathbf{A}^{-1}$