Math 3013

ANSWERS TO SECOND EXAM

November 3, 2020

1. Consider the vectors $\{[1,2,1,1,0]\ ,\ [0,1,2,1,1]\ ,\ [-1,-1,1,0,1]\ ,\ [-1,0,3,1,2]\} \in \mathbb{R}^5$

$$\begin{aligned}
\begin{bmatrix} 1 & 0 & -3 & -1 & -2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} &= R.R.E.F.\,(\mathbf{A}) \\
&\Rightarrow \quad \text{not linearly independent} && \text{(a)} \\
\dim RowSp\,(\mathbf{A}) &= 2 && \text{(b)}
\end{aligned}$$

- 2. Definitions

3. Given that the matrix $\mathbf{A} = \begin{bmatrix} 2 & 4 & 0 & 2 \\ -1 & -2 & 1 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

-

$$\begin{aligned}
\text{basis for row space} &= \{[1,2,0,1]\ ,\ [0,0,1,-2]\} && \text{(a)} \\
\text{basis for column space} &= \left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} , \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\} && \text{(b)} \\
\text{basis for } NullSp\,(\mathbf{A}) &= \{[-2,1,0,0]\ ,\ [-1,0,2,1]\} && \text{(c)} \\
rank &= 2 && \text{(d)}
\end{aligned}$$

4.

-

$$\begin{aligned}
\mathbf{A}_T &= = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} && \text{(a)} \\
\text{basis for Range}\,(T) &= \{[1,1,1]\ ,\ [2,1,0]\} && \text{(b)} \\
\text{basis for } \ker(T) &= \{[1,-1,1]\} && \text{(c)}
\end{aligned}$$

5. (15 pts) cofactor expansions $\Rightarrow$ $\det(\mathbf{A}) = -2$
6. (15 pts) row reduction method $\Rightarrow$ $\det(\mathbf{A}) = 2$