Math 3013
PRACTICE SECOND EXAM
Spring 2022

1. Define, precisely, the following notions (where V, W are to be regarded as general vector spaces). space V).

(a) (5 pts) a **subspace** of V is ...

(b) (5 pts) a **basis** for a vector space V is ...

(c) (5 pts) a **set of linearly independent vectors** in V is ...

(d) (5 pts) a **linear transformation** from a vector space V to vector space W is ...

2. (10 pts) Prove or disprove that the points on the circle $S = \{[x, y] \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ is a subspace of $\mathbb{R}^2$.

3. (10 pts) Let $W = span\left([1,1,1],[1,-2,1],[3,0,3]\right) \subset \mathbb{R}^3$. Find a basis for W

4. Consider the following matrix: $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

(a) (10 pts) Row reduce this matrix to reduced row echelon form

(b) (5 pts) Find a basis for the row space of **A**.

(c) (5 pts) Find a basis for the column space of **A**.

(d) (5 pts) Find a basis for the null space of **A**.

(e) (5 pts) What is the rank of **A**?

5. (10 pts) Let $\mathbf{A}$ be an $n \times m$ matrix. Show that the solution set of $\mathbf{Ax} = \mathbf{0}$ is a subspace of $\mathbb{R}^m$

6. Consider the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^2 : T([x_1, x_2, x_3, x_4]) = [x_1 - x_3 + 2x_4, 2x_2 - 2x_3 + x_4]$.

(a) (10 pts) Find the matrix $\mathbf{A}_T$ representating T:

(b) (5 pts) Find a basis for $range(T) \equiv \{\mathbf{y} \in \mathbb{R}^2 \mid \mathbf{y} = T(\mathbf{x}) \text{ for some } \mathbf{x} \in \mathbb{R}^4\}$.

(c) (5 pts) Find a basis for $ker(T) \equiv \{\mathbf{x} \in \mathbb{R}^4 \mid T(\mathbf{x}) = \mathbf{0}\}$