Math 3013

PRACTICE SECOND EXAM

1. Consider the vectors $\{[1,1,1,0],[1,0,1,1],[1,-1,1,2],[1,0,0,-1]\} \in \mathbb{R}^4$

(a) (10 pts) Determine if these vectors are linearly independent.

(b) (5 pts) What is the dimension of the subspace generated by these vectors?

2. Write the definitions (as stated in class) of the following notions. (5 pts each)

(a) (A subset of $\mathbb{R}^n$ that is) **closed under scalar multiplication**

(b) A subset of $\mathbb{R}^n$ that is) **closed under vector addition.**

(c) A **subspace** of $\mathbb{R}^n$.

(d) A **basis** for a subspace of $\mathbb{R}^n$.

(e) A set of **linearly independent vectors**

3. Given that the following matrix: $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & -1 \\ 1 & -3 & -1 & -3 \\ 2 & -2 & 0 & -2 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a) (5 pts) Find a basis for the row space of $\mathbf{A}$.

(b) (5 pts) Find a basis for the column space of $\mathbf{A}$.

(c) (5 pts) Find a basis for the null space of $\mathbf{A}$.

(d) (5 pts) What is the rank of $\mathbf{A}$?

4. Consider the following linear transformation: $T : \mathbb{R}^2 \to \mathbb{R}^4 : T\left([x_1, x_2\right) = [x_2, x_1, x_1 - x_2, x_1 + x_2]$.
(a) (10 pts) Find a matrix that represents T.

(b) (5 pts) Find a basis for the range of T.

(c) (5 pts) Find a basis for the kernel of T.

5. Compute the determinants of the following matrices (5 pts each)

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 3 & 1 & 0 \\ 1 & 2 & 3 & 2 & 1 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$