Lecture 14 : Review for Exam 1

Math 3013
Oklahoma State University

February 16, 2022

Agenda

1. Quick Overview of Material to be Covered on the 1st Exam
2. Practice Exam

Quick Overview of Material on First Midterm

I. Vectors in $\mathbb{R}^n$

A. Vector Addition $\quad + : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$

B. Scalar Multiplication $\quad * : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$

C. Linear Combinations of Vectors : e.g. $\alpha \mathbf{v} + \beta \mathbf{u} + \cdots$

D. Dot Product $\quad \cdot : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$

II. Geometry of Vector Spaces

A. Points, Lines, Planes and Hyperplanes

$$\mathcal{H} = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{x} = \mathbf{p}_0 + t_1\mathbf{v}_1 + t_2\mathbf{v}_2 \cdots t_k\mathbf{v}_k ; t_1, t_2, \ldots, t_k \in \mathbb{R}\}$$

B. Solutions Sets of Linear Equations are (intersections of) Hyperplanes

$$a_1x_1 + \cdots + a_{m-1}x_{m-1} + a_m x_m = b$$

$$\Rightarrow \quad \mathbf{x} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \frac{b}{a_m} \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ \vdots \\ 0 \\ -\frac{a_1}{a_m} \end{bmatrix} + \cdots + x_{m-1} \begin{bmatrix} 0 \\ \vdots \\ 1 \\ -\frac{a_{m-1}}{a_m} \end{bmatrix}$$

Overview, Cont'd

III. Matrices and Matrix Algebra

A. Matrices and Linear Systems : Augmented Matrices

$$\left.\begin{array}{l} a_{11}x_1 + \cdots + a_{1m}x_m = b_1 \\ \vdots \\ a_{n1}x_1 + \cdots + a_{nm}x_m = b_n \end{array}\right\} \Longleftrightarrow \left[\begin{array}{ccc|c} a_{11} & \cdots & a_{1m} & b_1 \\ \vdots & \ddots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nm} & b_n \end{array}\right]$$

B. Matrix Multiplication

C. Matrix Addition and Scalar Multiplication

D. The Transpose of a Matrix

Overview, Cont'd

IV. Solving Systems of Linear Equations

A. Elementary Operations on Systems of Equations

B. Augmented Matrices and Elementary Row Operations

C. Row-Echelon Form

D. Reduced Row-Echelon Form

F. Solving Linear Systems

- convert to augmented matrix $[\mathbf{A} \mid \mathbf{b}]$
- row reduce to R.E.F.
- row reduce further to R.R.E.F.
- identify fixed and free variables in solution
- write down solution set as a hyperplane

Overview, Cont'd

V. Inverses of Square Matrices
   A. Definition and Properties of Matrix Inverses
   B. Elementary Matrices
   C. Calculating Matrix Inverses
      - form adjoined matrix $[\mathbf{A} \mid \mathbf{I}]$
      - row reduce $[\mathbf{A} \mid \mathbf{I}]$ to R.R.E.F.
      - if L.H.S. of R.R.E.F matrix is the identity matrix $\mathbf{I}$, then the R.H.S. is $\mathbf{A}^{-1}$

        if not, $\mathbf{A}$ has no inverse
   D. Fundamental Theorem of Matrix inverses and $n \times n$ linear systems
      - $\mathbf{A}$ has an inverse.
      - $\mathbf{Ax} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$
      - the only solution of $\mathbf{Ax} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$
      - the R.R.E.F. of $\mathbf{A}$ is the identity matrix $\mathbf{I}$
      - $\mathbf{A}$ is a product of elementary matrices