Math 3013
WebAssign Problems #3

1. Let $\mathbf{A} = \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 & -3 & -1 \\ 4 & -2 & -5 \end{bmatrix}$. Compute $\mathbf{AB}$.

   - $$\begin{aligned} \mathbf{AB} &= \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 3 & -3 & -1 \\ 4 & -2 & -5 \end{bmatrix} \\ &= \begin{bmatrix} (3,0)\cdot(3,4) & (3,0)\cdot(-3,-2) & (3,0)\cdot(-1,-5) \\ (-1,5)\cdot(3,4) & (-1,5)\cdot(-3,-2) & (-1,5)\cdot(-1,-5) \end{bmatrix} \\ &= \begin{bmatrix} 9+0 & -9+0 & -3+0 \\ -3+20 & 3-10 & 1-25 \end{bmatrix} \\ &= \begin{bmatrix} 9 & -9 & -3 \\ 17 & -7 & -24 \end{bmatrix} \end{aligned}$$

2. Let $\mathbf{B} = \begin{bmatrix} 3 & -5 & 1 \\ 0 & 5 & 2 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 0 & -2 \\ -6 & 1 \end{bmatrix}$. Compute $\mathbf{BD}$.

   - $$\mathbf{BD} = \begin{bmatrix} 3 & -5 & 1 \\ 0 & 5 & 2 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ -6 & 1 \end{bmatrix}$$

     This product is undefined – since the number of columns of the first factor does not equal the number of rows of the second factor (i.e. $3 \neq 2$).

     So the correct answer is DNE (which stands for Does Not Exist).

3. Let $\mathbf{A} = \begin{bmatrix} 4 & -5 \\ -1 & 2 \end{bmatrix}$, $\mathbf{E} = \begin{bmatrix} 4 & 2 \end{bmatrix}$, and $\mathbf{F} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Compute $\mathbf{E}(\mathbf{AF})$.

   - We have
     $$\mathbf{AF} = \begin{bmatrix} 4 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} (4)(-1)+(-5)(2) \\ (-1)(-1)+(2)(2) \end{bmatrix} = \begin{bmatrix} -14 \\ 5 \end{bmatrix}$$
     and so
     $$\mathbf{E}(\mathbf{AF}) = \begin{bmatrix} 4 & 2 \end{bmatrix} \begin{bmatrix} -14 \\ 5 \end{bmatrix} = [(4)(-14)+(2)(5)] = [-46]$$

4. Let $\mathbf{E} = \begin{bmatrix} 12 & 4 \end{bmatrix}$ and $\mathbf{F} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$. Compute $\mathbf{EF}$.

   - $$\mathbf{EF} = \begin{bmatrix} 12 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = [(12)(-1)+(4)(3)] = [0]$$

5. Let $\mathbf{A} = \begin{bmatrix} 4 & 0 \\ -1 & 5 \end{bmatrix}$. Compute $\mathbf{A}^3$.

   - We have
     $$\begin{aligned} \mathbf{A}^2 &= \begin{bmatrix} 4 & 0 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} (4)(4)+(0)(-1) & (4)(0)+(0)(5) \\ (-1)(4)+(5)(-1) & (-1)(0)+(5)(5) \end{bmatrix} \\ &= \begin{bmatrix} 16 & 0 \\ -9 & 25 \end{bmatrix} \end{aligned}$$

and so

$$\begin{aligned}\mathbf{A}^2 &= \mathbf{A}\mathbf{A}^2 = \begin{bmatrix} 4 & 0 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 16 & 0 \\ -9 & 25 \end{bmatrix} = \begin{bmatrix} (4)(16)+(0)(-9) & (4)(0)+(0)(25) \\ (-1)(16)+(5)(-9) & (-1)(0)+(5)(25) \end{bmatrix} \\ &= \begin{bmatrix} 64 & 0 \\ -61 & 125 \end{bmatrix}\end{aligned}$$

6. Let $\mathbf{A} = \begin{bmatrix} 6 & 0 \\ -1 & 3 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 0 & -6 \\ -4 & 1 \end{bmatrix}$. Compute $\mathbf{A} + 2\mathbf{D}$.

   - $$2\mathbf{D} = \begin{bmatrix} 0 & -12 \\ -8 & 2 \end{bmatrix}$$

     and so

     $$\mathbf{A} + 2\mathbf{D} = \begin{bmatrix} 6 & 0 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & -12 \\ -8 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -12 \\ -9 & 5 \end{bmatrix}$$

7. Let $\mathbf{B} = \begin{bmatrix} 6 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 5 & 6 \end{bmatrix}$. Compute $\mathbf{B} - \mathbf{C}$.

   - $\mathbf{B}$ is a 2×3 matrix, and $\mathbf{C}$ is a 3×2 matrix. Matrices of different types can not be added together, so the correct answer is DNE (Does Not Exist).

8. Let $\mathbf{B} = \begin{bmatrix} 6 & -5 & 2 \\ 0 & 5 & 4 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 6 & 5 \\ 3 & 1 \\ 4 & 2 \end{bmatrix}$. Compute $\mathbf{C} - \mathbf{B}^T$.

   - $\mathbf{B}^T$ is the transpose of the matrix $\mathbf{B}$, and is obtained by converting the rows of $\mathbf{B}$ into columns (or the columns of $\mathbf{B}$ into rows). Thus,

     $$\mathbf{B}^T = \begin{bmatrix} 6 & 0 \\ -5 & 5 \\ 1 & 4 \end{bmatrix}$$

     and so

     $$\mathbf{C} - \mathbf{B}^T = \begin{bmatrix} 6 & 5 \\ 3 & 1 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ -5 & 5 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 8 & -4 \\ 3 & -2 \end{bmatrix}$$