Math 4023
Homework Set 5

1. Show that each of the following subsets is not compact by describing an open cover for it that does not have a finite subcover

   (a) $S = [1, 3)$.
   (b) $S = \mathbb{N}$.
   (c) $S = \left\{ \frac{1}{n} \mid n \in \mathbb{N} \right\}$

2. Prove that the intersection of any collection of compact sets is compact.

   (a) Prove the if S and T are compact subsets of $\mathbb{R}$ then $S \cup T$ is compact.
   (b) Find an infinite collection $\{S_n \mid n \in \mathbb{N}\}$ of compact subsets of (R) such that

   $$bigcup_{n \in \mathbb{N}} S_n$$

   is not compact.

3. Let $\mathcal{F}$ be a collection of disjoint open subsets of $\mathbb{R}$. Prove that $\mathcal{F}$ is countable.

4. If S is a compact subset of $\mathbb{R}$ and T is a closed subset of S, then T is compact.

   (a) Prove this using the definition of compactness.
   (b) Prove this using the Heine-Borel theorem.