Math 4023
Homework Set 6

1. Use the definition of convergence to show

    (a) For any real number k, $\lim_{n\to\infty}(k/n) = 0$

    (b) For any real number $k > 0$, $\lim_{n\to\infty}(1/n^k) = 0$

    (c) $\lim(3n+1)/(n+2) = 3$

2. Show that the following sequences are divergent.

    (a) $a_n = 2n$

    (b) $b_n = (-1)^n$

3. Suppose that $\lim s_n = 0$ and that t_n is a bounded sequence. Prove that $\lim(s_n t_n) = 0$.

4. Suppose that $\lim s_n = s > 0$. Prove that there exists an $N \in \mathbb{R}$ such that $s_n > 0$ for all $n > N$.

5. Prove that if x is an accumulation point of a set S if and only if there exists a sequence (s_n) of points in $S \setminus \{x\}$ such that (s_n) converges to x.

6. Prove that a set is closed if and only if whenever (s_n) is a convergent sequence of point in S, it follows that $\lim s_n$ is in S.

7. Determine whether the following sequences are convergent or divergent.

    (a) $s_n = (3 - 2n)/(1 + n)$

    (b) $s_n = (-1)^n/(n+3)$

    (c) $s_n = (-1)^n \, n/(2n - 1)$

    (d) $s_n = (2^{3n})/(3^{2n})$

    (e) $s_n = n^2/2^n$