

APPENDIX A. CBC DIAGRAMS FOR THE NILPOTENT ORBITS OF EXCEPTIONAL LIE ALGEBRAS

CBCP Diagrams for E_6

1	$\mathbf{0}$	$\begin{array}{c} \circ \\ \\ \circ - \circ - \circ - \circ - \circ \end{array}$	12	$D_4(a_1)$	$\begin{array}{c} \bullet \\ \\ \circ - \bullet - * - \bullet - \circ \end{array}$
2	A_1	$\begin{array}{c} \circ \\ \\ \bullet - \circ - \circ - \circ - \circ \end{array}$	13	A_4	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - \bullet - \circ - \circ \end{array}$
3	$2A_1$	$\begin{array}{c} \bullet \\ \\ \bullet - \circ - \circ - \circ - \circ \end{array}$	14	D_4	$\begin{array}{c} \bullet \\ \\ \circ - \bullet - \bullet - \bullet - \circ \end{array}$
4	$3A_1$	$\begin{array}{c} \bullet \\ \\ \bullet - \circ - \circ - \bullet - \circ \end{array}$	15	$A_4 + A_1$	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - \bullet - \circ - \bullet \end{array}$
5	A_2	$\begin{array}{c} \circ \\ \\ \bullet - \bullet - \circ - \circ - \circ \end{array}$	16	A_5	$\begin{array}{c} \circ \\ \\ \bullet - \bullet - \bullet - \bullet - \bullet \end{array}$
6	$A_2 + A_1$	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - \circ - \circ - \circ \end{array}$	17	$D_5(a_1)$	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - * - \bullet - \circ \end{array}$
7	$2A_2$	$\begin{array}{c} \circ \\ \\ \bullet - \bullet - \circ - \bullet - \bullet \end{array}$	18	$E_6(a_3)$	$\begin{array}{c} * \\ \\ \bullet - * - \bullet - * - \bullet \end{array}$
8	$A_2 + 2A_1$	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - \circ - \bullet - \circ \end{array}$	19	D_5	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - \bullet - \bullet - \circ \end{array}$
9	A_3	$\begin{array}{c} \circ \\ \\ \bullet - \bullet - \bullet - \circ - \circ \end{array}$	20	$E_6(a_1)$	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - * - \bullet - \bullet \end{array}$
10	$2A_2 + A_1$	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - \circ - \bullet - \bullet \end{array}$	21	E_6	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - \bullet - \bullet - \bullet \end{array}$
11	$A_3 + A_1$	$\begin{array}{c} \bullet \\ \\ \bullet - \circ - \bullet - \bullet - \circ \end{array}$			

CBCP Diagrams for E_7

1	$\mathbf{0}$		24	$D_4 + A_1$	
2	A_1		25	$A_4 + A_1$	
3	$2A_1$		26	$D_5(a_1)$	
4	$(3A_1)''$		27	$A_4 + A_2$	
5	$(3A_1)'$		28	$(A_5)'$	
6	A_2		29	$A_5 + A_1$	
7	A_1		30	$D_5(a_1) + A_1$	
8	$A_2 + A_1$		31	$D_6(a_2)$	
9	$A_2 + 2A_1$		32	$E_6(a_3)$	
10	A_3		33	D_5	
11	$2A_2$		34	$E_7(a_5)$	
12	$A_2 + 3A_1$		35	A_6	
13	$(A_3 + A_1)''$		36	$D_5 + A_1$	
14	$2A_2 + A_1$		37	$D_6(a_1)$	
15	$(A_3 + A_1)'$		38	$E_7(a_4)$	
16	$D_4(a_1)$		39	D_6	
17	$A_3 + 2A_1$		40	$E_6(a_1)$	
18	D_4		41	E_6	
19	$D_4(a_1) + A_1$		42	$E_7(a_3)$	
20	$A_3 + A_2$		43	$E_7(a_2)$	
21	A_4		44	$E_7(a_1)$	
22	$A_3 + A_2 + A_1$		45	E_7	
23	$(A_5)''$				

CBCP Diagrams for E_8

1	$\mathbf{0}$		36	$A_5 + A_1$	
2	A_1		37	$D_5(a_1) + A_2$	
3	$2A_1$		38	$D_6(a_2)$	
4	$3A_1$		39	$E_6(a_3) + A_1$	
5	A_2		40	$E_7(a_5)$	
6	$4A_1$		41	$D_5 + A_1$	
7	$A_2 + A_1$		42	$E_8(a_7)$	
8	$A_2 + 2A_1$		43	A_6	
9	A_3		44	$D_6(a_1)$	
10	$A_2 + 3A_1$		45	$A_6 + A_1$	
11	$2A_2$		46	$E_7(a_4)$	
12	$2A_2 + A_1$		47	$E_7(a_4)$	
13	$A_3 + A_1$		48	$D_5 + A_2$	
14	$D_4(a_1)$		49	D_6	
15	D_4		50	E_6	
16	$2A_2 + 2A_1$		51	$D_7(a_2)$	
17	$A_3 + 2A_1$		52	A_7	
18	$D_4(a_1) + A_1$		53	$E_6(a_1) + A_1$	
19	$A_3 + A_2$		54	$E_7(a_3)$	
20	A_4		55	$E_8(b_6)$	
21	$A_3 + A_2 + A_1$		56	$D_7(a_1)$	
22	$D_4 + A_1$		57	$E_6 + A_1$	
23	$D_4(a_1) + A_2$		58	$E_7(a_2)$	
24	$A_4 + A_1$		59	$E_8(a_6)$	
25	$2A_3$		60	D_7	
26	$D_5(a_1)$		61	$E_8(b_5)$	
27	$A_4 + 2A_1$		62	$E_7(a_1)$	
28	$A_4 + A_2$		63	$E_8(a_5)$	
29	A_5		64	$E_8(b_4)$	
30	$D_5(a_1) + A_1$		65	E_7	
31	$A_4 + A_2 + A_1$		66	$E_8(a_4)$	
32	$D_4 + A_2$		67	$E_8(a_3)$	
33	$E_6(a_3)$		68	$E_8(a_2)$	
34	D_5		69	$E_8(a_1)$	
35	$A_4 + A_3$		70	E_8	

CBCP Diagrams for F_4

1	$\mathbf{0}$	$\circ - \circ \Rightarrow \circ - \circ$
2	A_1	$\bullet - \circ \Rightarrow \circ - \circ$
3	$\tilde{A}_1$	$\circ - \circ \Rightarrow \bullet - \circ$
4	$A_1 + \tilde{A}_1$	$\bullet - \circ \Rightarrow \bullet - \circ$
5	A_2	$\bullet - \bullet \Rightarrow \circ - \circ$
6	$\tilde{A}_2$	$\circ - \circ \Rightarrow \bullet - \bullet$
7	$A_2 + \tilde{A}_1$	$\bullet - \bullet \Rightarrow \circ - \bullet$
8	B_2	$\circ - \bullet \Rightarrow \bullet - \circ$
9	$\tilde{A}_2 + A_1$	$\bullet - \circ \Rightarrow \bullet - \bullet$
10	$C_3(a_1)$	$\circ - \bullet \Rightarrow * - \bullet$
11	$F_4(a_3)$	$* - \bullet \Rightarrow * - *$
12	B_3	$\bullet - \bullet \Rightarrow \bullet - \circ$
13	C_3	$\circ - \bullet \Rightarrow \bullet - \bullet$
14	$F_4(a_2)$	$* - \bullet \Rightarrow * - \bullet$
15	$F_4(a_1)$	$\bullet - \bullet \Rightarrow * - \bullet$
16		$\bullet - \bullet \Rightarrow \bullet - \bullet$

CBCP Diagrams for G_2

1	$\mathbf{0}$	$\circ \Leftarrow \circ$
2	A_1	$\circ \Leftarrow \bullet$
3	$\tilde{A}_1$	$\bullet \Leftarrow \circ$
4	$G_2(a_1)$	$* \Leftarrow \bullet$
5	G_3	$\bullet \Leftarrow \bullet$

APPENDIX B. CBC DIAGRAMS AND PARTITIONS FOR DISTINGUISHED NILPOTENT ORBITS OF CLASSICAL GROUPS

<i>BC label</i>	<i>CBC diagram</i>	<i>partition</i>
A_n	$\bullet - \bullet \cdots \cdots \cdots \bullet - \bullet$	$[n+1]$

<i>BC label</i>	<i>CBC diagram</i>	<i>partition</i>
B_2	$\bullet \Rightarrow \bullet$	$[5]$
B_3	$\bullet - \bullet \Rightarrow \bullet$	$[7]$
B_4	$\bullet - \bullet - \bullet \Rightarrow \bullet$	$[9]$
$B_4(a_2)$	$\bullet - * - \bullet \Rightarrow *$	$[5, 3, 1]$
B_5	$\bullet - \bullet - \bullet - \bullet \Rightarrow \bullet$	$[11]$
$B_5(a_2)$	$\bullet - \bullet - * - \bullet \Rightarrow *$	$[7, 3, 1]$
B_6	$\bullet - \bullet - \bullet - \bullet - \bullet \Rightarrow \bullet$	$[13]$
$B_6(a_2)$	$\bullet - \bullet - \bullet - * - \bullet \Rightarrow *$	$[9, 3, 1]$
$B_6(a_3)$	$\bullet - * - \bullet - * - \bullet \Rightarrow *$	$[7, 5, 1]$
B_7	$\bullet - \bullet - \bullet - \bullet - \bullet - \bullet \Rightarrow \bullet$	$[15]$
$B_7(a_2)$	$\bullet - \bullet - \bullet - \bullet - * - \bullet \Rightarrow *$	$[11, 3, 1]$
$B_7(a_3)$	$\bullet - \bullet - * - \bullet - * - \bullet \Rightarrow *$	$[9, 5, 1]$
$B_7(a_4)$	$\bullet - * - \bullet - * - * - \bullet \Rightarrow *$	$[7, 5, 3]$
B_8	$\bullet - \bullet - \bullet - \bullet - \bullet - \bullet - \bullet \Rightarrow \bullet$	$[17]$
$B_8(a_2)$	$\bullet - \bullet - \bullet - \bullet - \bullet - * - \bullet \Rightarrow *$	$[13, 3, 1]$
$B_8(a_3)$	$\bullet - \bullet - \bullet - * - \bullet - * - \bullet \Rightarrow *$	$[11, 5, 1]$
$B_8(a_4)$	$\bullet - * - \bullet - * - \bullet - * - \bullet \Rightarrow *$	$[9, 7, 1]$
$B_8(b_4)$	$\bullet - \bullet - * - \bullet - * - * - \bullet \Rightarrow *$	$[7, 5, 3]$

<i>BC label</i>	<i>CBC diagram</i>	<i>partition</i>
C_2	$\bullet \Leftarrow \bullet$	$[4]$
C_3	$\bullet - \bullet \Leftarrow \bullet$	$[6]$
$C_3(a_1)$	$\bullet - * \Leftarrow \bullet$	$[4, 2]$
C_4	$\bullet - \bullet - \bullet \Leftarrow \bullet$	$[8]$
$C_4(a_1)$	$\bullet - \bullet - * \Leftarrow \bullet$	$[6, 2]$
C_5	$\bullet - \bullet - \bullet - \bullet \Leftarrow \bullet$	$[10]$
$C_5(a_1)$	$\bullet - \bullet - \bullet - * \Leftarrow \bullet$	$[8, 2]$
$C_5(a_2)$	$\bullet - * - \bullet - * \Leftarrow \bullet$	$[6, 4]$
C_6	$\bullet - \bullet - \bullet - \bullet - \bullet \Leftarrow \bullet$	$[12]$
$C_6(a_1)$	$\bullet - \bullet - \bullet - \bullet - * \Leftarrow \bullet$	$[10, 2]$
$C_6(a_2)$	$\bullet - \bullet - * - \bullet - * \Leftarrow \bullet$	$[8, 4]$
$C_6(a_3)$	$\bullet - * - \bullet - * - * \Leftarrow \bullet$	$[6, 4, 2]$
C_7	$\bullet - \bullet - \bullet - \bullet - \bullet - \bullet \Leftarrow \bullet$	$[14]$
$C_7(a_1)$	$\bullet - \bullet - \bullet - \bullet - \bullet - * \Leftarrow \bullet$	$[12, 2]$
$C_7(a_2)$	$\bullet - \bullet - \bullet - * - \bullet - * \Leftarrow \bullet$	$[10, 4]$
$C_7(a_3)$	$\bullet - * - \bullet - * - \bullet - * \Leftarrow \bullet$	$[8, 6]$
$C_7(b_3)$	$\bullet - \bullet - * - \bullet - * - * \Leftarrow \bullet$	$[8, 4, 2]$
C_8	$\bullet - \bullet - \bullet - \bullet - \bullet - \bullet - \bullet \Leftarrow \bullet$	$[16]$
$C_8(a_1)$	$\bullet - \bullet - \bullet - \bullet - \bullet - \bullet - * \Leftarrow \bullet$	$[14, 2]$
$C_8(a_2)$	$\bullet - \bullet - \bullet - \bullet - * - \bullet - * \Leftarrow \bullet$	$[12, 4]$
$C_8(a_3)$	$\bullet - \bullet - * - \bullet - * - \bullet - * \Leftarrow \bullet$	$[10, 6]$
$C_8(b_3)$	$\bullet - \bullet - \bullet - * - \bullet - * - * \Leftarrow \bullet$	$[10, 4, 2]$
$C_8(a_4)$	$\bullet - * - \bullet - * - \bullet - * - * \Leftarrow \bullet$	$[8, 6, 2]$

<i>BC label</i>	<i>CBC diagram</i>	<i>partition</i>
D_2		$[3, 1]$
D_3		$[5, 1]$
D_4		$[7, 1]$
$D_4(a_1)$		$[5, 3]$
D_5		$[9, 1]$
$D_5(a_1)$		$[7, 3]$
D_6		$[11, 1]$
$D_6(a_1)$		$[9, 3]$
$D_6(a_2)$		$[7, 5]$
D_7		$[13, 1]$
$D_7(a_1)$		$[11, 3]$
$D_7(a_2)$		$[9, 5]$
D_8		$[15, 1]$
$D_8(a_1)$		$[13, 3]$
$D_8(a_2)$		$[11, 5]$
$D_8(a_3)$		$[9, 7]$
$D_8(a_5)$		$[7, 5, 3, 1]$