HC-Cells, Nilpotent Orbits, Primitive Ideals and Weyl Group Representations

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Setting

- G : set of real points of a connected, complex algebraic group $G_{\mathbb{C}}$ defined over $\mathbb{R}$
- $\widehat{G}_{adm} = \{\text{irr. adm. reps of } G\}$

Objective: Understand the organization of $\widehat{G}_{adm}$ in terms of algebraic invariants.

Setting

- $(\Phi, \Pi, \Phi^\vee, \Pi^\vee)$: root datum
- $G_{\mathbb{C}}$: a connected, linear, complex algebraic group defined over $\mathbb{R}$
- G : set of real points of a connected, complex algebraic group $G_{\mathbb{C}}$ defined over $\mathbb{R}$
- $\widehat{G}_{adm} = \{\text{irr adm reps of } G\}$

Objective: Understand the organization of $\widehat{G}_{adm}$ in terms of algebraic invariants.

$$\text{irr adm reps} \longleftrightarrow \text{irr } (\mathfrak{g}, K)\text{-modules} \longleftrightarrow \text{Langlands parms}$$

First Reduction: $\widehat{G}_{adm,\lambda} = \{\text{irr. adm. reps of inf char } \lambda\}$

(w/o loss of generality by Borho-Jantzen-Zuckerman translation principle)

Assumption: λ is assumed to be regular and integral

Approach: W-graph structure of $\widehat{G}_{adm,\lambda} \longrightarrow$ algebraic invariants

Implicit Theme: Atlas software makes these ideas computable.

Blocks of irreducible Harish-Chandra modules

"Under the hood" of the `atlas` software is a parameterization of $\widehat{G}_{adm,\rho}$ in terms of pairs

$$(x, y) \in K\backslash G/B \times K^{\vee}\backslash G^{\vee}/B^{\vee}$$

(There is also a certain compatibility condition between x and y.)

Definition

A **block** of representations is set of representations for which the pairs (x, y) range over $K\backslash G/B \times K^{\vee}\backslash G^{\vee}/B^{\vee}$ corresponding to fixed real forms of G and $G^{\vee}$.

Atlas's representation theoretical computations take place block by block.

Example: the blocks of E8

Below is a table is listing the number of elements in each "block" of E_8:

	e_8	$E_8(e_7, su(2))$	$E_8(\mathbb{R})$
e_8	0	0	1
$E_8(e_7, su(2))$	0	3150	73410
$E_8(\mathbb{R})$	1	73410	453060

The total number of equivalences classses irreducible Harish-Chandra modules of the split form $E_8(\mathbb{R})$ with infinitesimal character ρ is thus

$$1 + 73410 + 453060 = 526\,471$$

Atlas PoV : if you're going to look outside a particular block you may as well consider all the blocks of $G_{\mathbb{C}}$.

Cells of Harish-Chandra modules

Definition

Given two reps x, y in HC_λ, we say

$$x \rightsquigarrow y \quad \Longleftrightarrow \quad \exists \text{ f.d. rep } F \subset \bigoplus_{n=0}^{\infty} \mathfrak{g}^{\otimes^n} \quad \text{s.t.} \quad x \text{ occurs as subquotient of } y \otimes F$$

$$x \sim y \text{ if } x \rightsquigarrow y \text{ and } y \rightsquigarrow x$$

The equivalence classes for the relation $\sim$ are called **cells** (of HC-modules).

Blocks and Cells of HC modules: graph-theoretical formulation

Definition

Given $x, y \in HC_\lambda$, we say

$$x \rightarrow y \quad \Longrightarrow \quad x \text{ occurs in } y \otimes \mathfrak{g}$$

The relation "$\rightarrow$" gives HC_λ the structure of a directed graph.

" $\rightsquigarrow$ "	$\longleftrightarrow$	transitive closure of " $\rightarrow$ "
cells of reps	$\longleftrightarrow$	strongly connected components of graph
blocks of reps	$\longleftrightarrow$	connected components of graph

The `atlas` software explicitly computes this digraph structure as a *by-product* of its computation of the *KLV*-polynomials.

W-graphs

In fact, atlas's KLV polynomial computations endow HC_λ with even more elaborate graph structure.

Definition

Let B be a block of irr HC modules of inf char λ.
The W-**graph** of B is the weighted digraph where:

- the **vertices** are the elements $x \in B$
- there is an **edge** $x \to y$ of **multiplicity** m between two vertices if

$$\text{coefficient of } q^{(|x|-|y|-1)/2} \text{ in } P_{x,y}(q) = m \neq 0$$

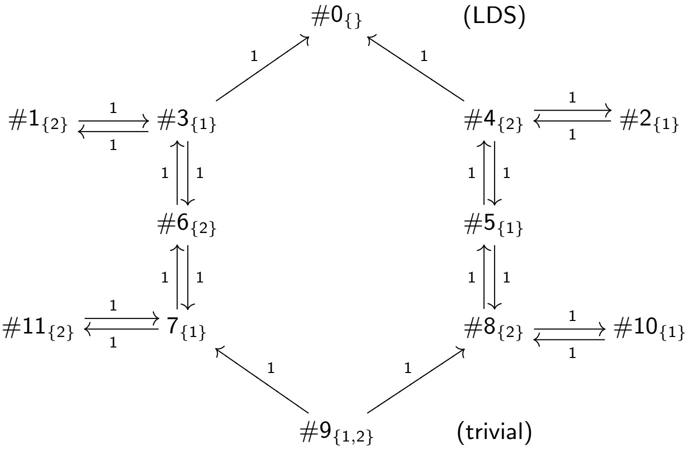
- there is assigned to each vertex x a subset $\tau(x)$ of the set of simple roots of $\mathfrak{g}$, the **descent set** of x.

Below is an example of the (annotated) output of `wgraph` for the big block of G_2.

block element	descent set	edge vertices, multiplicities
0	{}	{}
1	{2}	{(3,1)}
2	{1}	{(4,1)}
3	{1}	{(0,1),(1,1),(6,1)}
4	{2}	{(0,1),(2,1),(5,1)}
5	{1}	{(4,1),(8,1)}
6	{2}	{(3,1),(7,1)}
7	{1}	{(6,1),(11,1)}
8	{2}	{(5,1),(10,1)}
9	{1,2}	{(7,1),(8,1)}
10	{1}	{(8,1)}
11	{2}	{(7,1)}

Example cont'd: the W-graph of $G2$

The W-graph for this block thus looks like



Note that there are four cells: $\{\#0\}$, $\{\#1, \#3, \#6, \#7, \#11\}$, $\{\#2, \#4, \#5, \#8, \#10\}$, and $\{\#9\}$.

Definition

Let V be an irreducible $U(\mathfrak{g})$-module.

$$Ann(V) := \{X \in U(\mathfrak{g}) \mid Xv = 0 \quad , \quad \forall\, v \in V\}$$

is a two-sided ideal in $U(\mathfrak{g})$. It is called the *primitive ideal* in $U(\mathfrak{g})$ attached to V.

Fact: $Ann(V) = Ann(V') \Longrightarrow inf\ ch\ V = inf\ ch\ V'$

The correspondence

$$HC_\lambda \to Prim(\mathfrak{g})_\lambda : x \longmapsto Ann(x)$$

is often one-to-one, but generally speaking, several-to-one.

$\Rightarrow$ a fairly fine grained-partitioning of HC_λ

Nilpotent Orbits

$U(\mathfrak{g})$ is naturally filtered according to

$$U^n(\mathfrak{g}) = \{X \in U(\mathfrak{g}) \mid X = \text{product of} \leq n \text{ elements of } \mathfrak{g}\}$$

The graded algebra

$$gr(U(\mathfrak{g})) = \bigoplus_{n=0}^{\infty} U^n(\mathfrak{g}) / U^{n-1}(\mathfrak{g})$$

is well defined, and, in fact

$$gr(U(\mathfrak{g})) \approx S(\mathfrak{g})$$

Definition

Let J be a primitive ideal and set

$$\mathcal{V}(J) = \{\lambda \in \mathfrak{g}^* \mid \phi(\lambda) = 0 \quad \forall \phi \in gr(J)\}$$

The affine variety $\mathcal{V}(J)$ is called the *associated variety* of J.

Fundamental Facts:

$\mathcal{V}(J)$ is a closed, G-invariant subset of $\mathfrak{g}^*$.

In fact,

Theorem

$\mathcal{V}(J)$ is the Zariski closure of a single nilpotent orbit in $\mathfrak{g}^*$

Definition

Let $x \in HC_\lambda$ .The *nilpotent orbit attached to x* is the unique dense orbit $\mathcal{O}_x$ in $\mathcal{V}(Ann(x))$.

Lemma

If x, y belong to the same cell of HC-modules then $\mathcal{O}_x = \mathcal{O}_y$.

(ass variety doesn't change after tensoring with a finite-dim rep)

Different cells can share the same nilpotent orbit

$\rightsquigarrow$ rather coarse invariant of HC-modules

W acts naturally on the Grothendieck group $\mathbb{Z}HC_\lambda$ of irr HC modules of inf char λ via the "coherent continuation representation"

The W-representation carried by a cell is encoded in its W-graph.

The action of a simple reflection on cell rep corresponds to

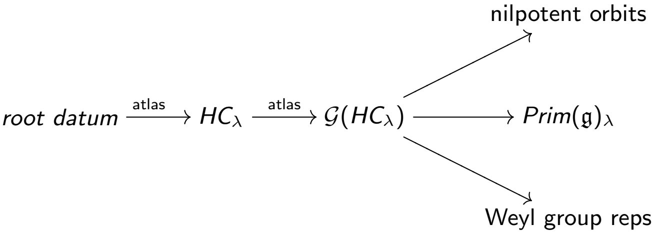
$$T_i x = \begin{cases} -x & i \in \tau(x) \\ x + \sum_{y : i \in \tau(y)} m_{y \to x} y & i \notin \tau(x) \end{cases}$$

The W-representation carried by a cell can be computed by evaluating

$$\chi_C(s_i \cdots s_j) = trace\,(T_i \cdots T_j)$$

on a representative $s_i \cdots s_j$ of each conjugacy class and then decomposing this character into a sum of irreducible characters (i.e., brute force is feasible)

Or via branching rules (Jackson-Noel) (spotting occurence of sign reps of Levi subgroups)


nilpotent orbits
root datum
atlas
HC_λ
atlas
$\mathcal{G}(HC_\lambda)$
$Prim(\mathfrak{g})_\lambda$
Weyl group reps

$\mathfrak{g} = Lie\,(G_{\mathbb{R}})_{\mathbb{C}}$; $\quad \mathfrak{h}$, a CSA for $\mathfrak{g}$;
$\Delta = \Delta\,(\mathfrak{h}, \mathfrak{g})$, roots of $\mathfrak{h}$ in $\mathfrak{g}$;
$\Pi \subset \Delta$, choice of simple roots in Δ;

G : adjoint group of $\mathfrak{g}$

$\mathcal{N}_{\mathfrak{g}}$: nilpotent cone in $\mathfrak{g}$ (identifying $\mathfrak{g}^*$ with $\mathfrak{g}$)

$\mathcal{S} \equiv$ {special nilpotent orbits} ($\leftrightarrow$ ass varieties of prim ideals of reg int inf char)

$d : G\backslash\mathcal{N}_{\mathfrak{g}} \longrightarrow G\backslash\mathcal{N}_{\mathfrak{g}}$: the Spaltenstein-Barbasch-Vogan duality map

d restricts to an involution on $image(d) \equiv \mathcal{S} \equiv$ set of **special** nilpotent orbits.

Definition

Let Γ be a subset of the simple roots. The corresponding **standard Levi subalgebra** $\mathfrak{l}_\Gamma$ is the subalgebra

$$\mathfrak{l}_\Gamma = \mathfrak{h} + \sum_{\alpha \in \mathbb{Z}\Gamma} \mathfrak{g}_\alpha$$

Definition

Let $\mathfrak{p} = \mathfrak{l} + \mathfrak{n}$ be the Levi decomposition of a parabolic subalgebra of $\mathfrak{g}$ and let $\mathcal{O}$ be nilpotent orbit in $\mathfrak{l}$.

$$ind_{\mathfrak{l}}^{\mathfrak{g}}(\mathcal{O}) := \text{unique dense orbit in } G \cdot (\mathcal{O} + \mathfrak{n})$$

When $\mathcal{O} = \mathbf{0}_{\mathfrak{l}}$, $ind_{\mathfrak{l}}^{\mathfrak{g}}(\mathcal{O})$ is called the **Richardson orbit** corresponding to $\mathfrak{l}$ (or to Γ if $\mathfrak{l} = \mathfrak{l}_\Gamma$).

Remark: Induction preserves “special-ness” and trivial orbits are always special $\Longrightarrow$ Richardson orbits are always special.

Not every special orbit is Richardson however.

Proposition. (Spaltenstein) A special orbit $\mathcal{O}$ is contained in the closure of a Richardson orbit $ind_{\mathfrak{l}_\Gamma}(\mathbf{0})$ if and only if the (special) W-rep attached to $\mathcal{O}$ contains the sign representation of W_Γ.

Theorem, (Vogan) Suppose C is a cell of H-C modules with associated special nilpotent orbit $\mathcal{O}_C$ and let $\mathfrak{l}_\Gamma$ be a (standard) Levi subalgebra of $\mathfrak{g}$. Then

$$\mathcal{O}_C \subset \overline{ind_{\mathfrak{l}_\Gamma}^{\mathfrak{g}}\left(\mathbf{0}_{\mathfrak{l}_\Gamma}\right)} \iff \exists\, x \in C \;\; s.t.\, \Gamma \subset \tau(x)$$

Upshot: tau invariants of a cell C constrain which Richardson orbit closures can contain $\mathcal{O}_C$

Set

$$\tau(C) \equiv \{\tau(x) \mid x \in C\} = \{\tau\text{-invariants of reps in } C\}$$

Empirical Fact: # distinct $\tau(C) =$ # special nilpotent orbits

Problem: A special orbit is not, in general, determined by the Richardson orbits that contain it.

Fact: every special orbit $\mathcal{O}$ **is** determined by

(i) the Richardson orbits that contain $\mathcal{O}$

(ii) the Richardon orbits that contain $d(\mathcal{O})$

David Vogan's Idea: The tau invariants of a cell might tell us which Richardson orbits contain $\mathcal{O}_C$ and which Richardson orbits contain $d\left(\mathcal{O}_C\right)$.

Fix Ψ : a set of *standard* Γ*'s*: a collection of $\Gamma \in 2^{\Pi}$ such that

$$\Psi \underset{\longleftrightarrow}{1:1} \{\text{conj classes of Levi subalgebras}\} \qquad \Gamma \mapsto G \cdot \mathfrak{l}_{\Gamma}$$

(E.g., choose std Γ's to be first in the lexicographic ordering of their W-conj class)

Partial Order Ψ as follows:

$$\Gamma \leq \Gamma' \Longleftrightarrow ind_{\mathfrak{l}_{\Gamma}}^{\mathfrak{g}}(\mathbf{0}) \subset \overline{ind_{\mathfrak{l}_{\Gamma'}}^{\mathfrak{g}}(\mathbf{0})}$$

Remark: this ordering tends to reverse the ordering by inclusion.

Definition: The **tau signature** of an H-C cell C is the pair

$$\tau_{sig}(C) \equiv \left(min\left(\tau(C) \cap \Psi\right) \ , \ min\left(\tau^{\vee}(C) \cap \Psi\right)\right)$$

Here $\tau^{\vee}(C)$ is the set of Π-complements of tau invariants in C:

$$\tau^{\vee}(C) = \{\Pi - \tau(x) \mid x \in C\}$$

Tau signatures for special orbits

Definition: Let $\mathcal{O}$ be a special orbit. The *tau signature* of $\mathcal{O}$ is the pair $(\tau(\mathcal{O}), \tau^{\vee}(\mathcal{O}))$ where

$$\tau(\mathcal{O}) = min\left\{\Gamma \in \Psi \mid \mathcal{O} \subset \overline{ind_{\mathfrak{l}_\Gamma}^{\mathfrak{g}}(\mathbf{0}_{\mathfrak{l}_\Gamma})}\right\}$$
$$\tau^{\vee}(\mathcal{O}) = min\left\{\Gamma \in \Psi \mid d(\mathcal{O}) \subset \overline{ind_{\mathfrak{l}_\Gamma}^{\mathfrak{g}}(\mathbf{0}_{\mathfrak{l}_\Gamma})}\right\}$$

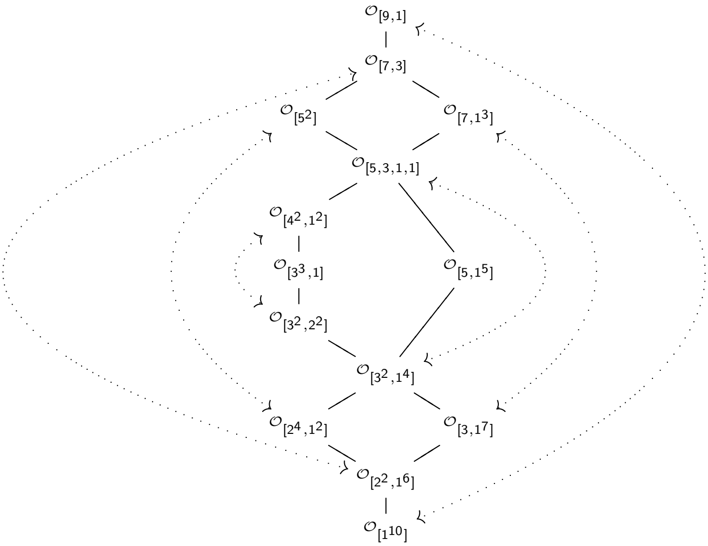
The point: we are using pairs of subsets of simple roots to tell us when a Richardson orbit closure can contain a special orbit (or its dual).

The same kind of pairs tells us when the orbit attached to a cell can be contained in Richardson orbit (or when the dual cell can be contained in the closure of Richardson orbit).

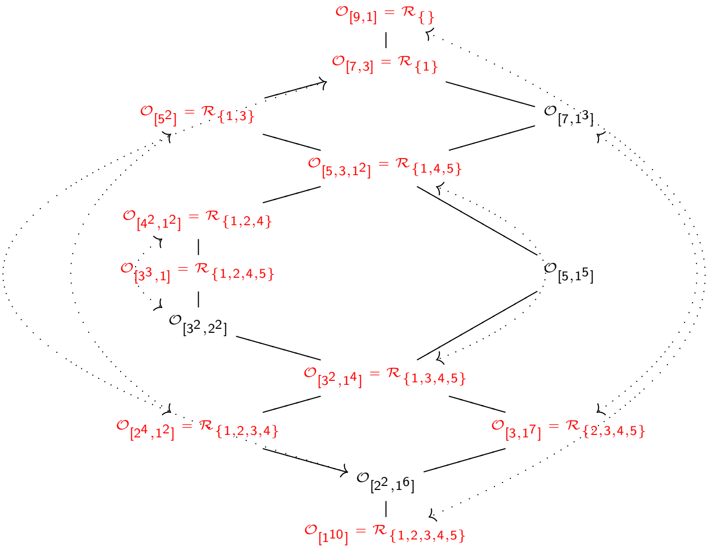
Corollary (to S-V criterion)

$$\mathcal{O}_C = \mathcal{O} \iff \tau_{sig}(C) = \tau_{sig}(\mathcal{O})$$

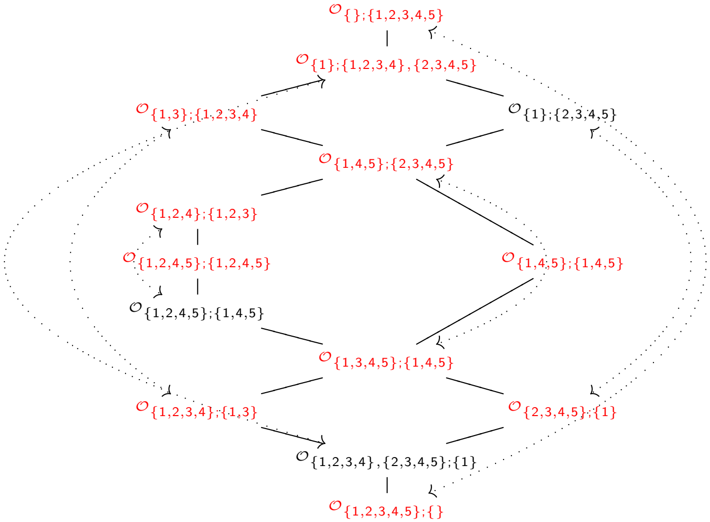
Example: Special Orbits of $D_5 \approx \mathfrak{so}(5,5)_{\mathbb{C}}$



Richardson Orbits of D_5



Tau Signatures of Special Orbits of D_5



Tau signatures for cells in the big block of $SO(5,5)$

- 365 representations with inf. char. ρ in big block
- 32 cells in the big block

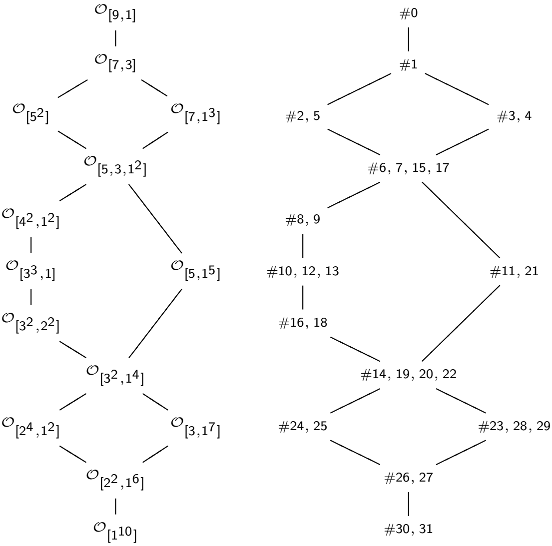
Output of `extract-cells`

```
// Individual cells.
// cell #0:
0[0]: {}

// cell #1:
0[1]: {2} --> 1,2
1[3]: {1} --> 0
2[5]: {3} --> 0,3,4
3[13]: {5} --> 2
4[14]: {4} --> 2
  *
  *
  *
// cell #29:
0[328]: {1,2,4,5} --> 2,3
1[340]: {2,3,4,5} --> 2
2[358]: {1,3,4,5} --> 0,1
3[364]: {1,2,3} --> 0

// cell #30:
0[353]: {1,2,3,4,5}

// cell #31:
0[357]: {1,2,3,4,5}
```

cell #	tau signature
0	{} , {1,2,3,4,5}
1	{1} , {1,2,3,4}
2	{1} , {2,3,4,5}
3	{1,3} , {1,3,4,5}
*	*
*	*
*	*
28	{2,3,4,5} , {1}
29	{2,3,4,5} , {1}
30	{1,2,3,4,5} , {}
31	{1,2,3,4,5} , {}

Each of these coincides with the tau signature of a particular nilpotent orbit.

Cell-Orbit Correspondences for $SO(5,5)$



More Generally:

Exceptional Groups: tables by Spaltenstein list induced orbits and Hasse diagrams.

Tau signatures of special orbits can be done by hand.

1. Use Spaltenstein's tables to figure out which special orbits are Richardson orbits and to identify the std Γ's corresponding to the corresponding Levi subalgebra.

2. Place the Richardson orbits in the Hasse diagram of special orbits, and then figure out the Γ parameters of the minimal Richardson orbits that contain a given special orbit and the minimal Richardson orbits that contain its Spaltenstein dual

Even E_8 can be done by hand.

Classical Groups:

Partition classification $\longrightarrow$ closure relations

Just need to

- which partitions correspond to special orbits (easy recipes in Collingwood-McGovern)
- use dominance ordering of partitions to partial order special orbits
- use formulas in [C-M] to determine partitions corresponding to Richardson orbits for each $\Gamma \in \Psi$. Place these in the Hasse diagram of special orbits and at the same time partial order Ψ.
- Use the partial ordering of Ψ to ascribe tau signatures to cells (employing `atlas` data)
- match orbit tau sigs to cell tau sigs

Standard modules and Irr HC-modules arise naturally in the study of $\widehat{G}_{\mathbb{R},adm}$

Verma and Irr HW modules much more convenient family for discussing primitive ideals.

Set

$\mathfrak{b} = \mathfrak{h} + \mathfrak{n}$: Borel subalgebra of $\mathfrak{g}$ $\qquad \rho = \frac{1}{2}\sum_{\alpha \in \Delta^+(\mathfrak{g},\mathfrak{h})} \alpha$

Theorem

Let $\lambda \in \mathfrak{h}^*$ and let $M(\lambda)$ denote the Verma module of highest weight $\lambda - \rho$; i.e., the left $U(\mathfrak{g})$-module

$$M(\lambda) := U(\mathfrak{g}) \otimes_{U(\mathfrak{b})} \mathbb{C}_{\lambda-\rho}$$

Then
(i) The Verma module $M(\lambda)$ has a unique irreducible quotient module $L(\lambda)$ which is of highest weight $\lambda - \rho$.
(ii) Every irreducible highest weight module is isomorphic to some $L(\lambda)$.

Duflo's theorem

Theorem (Duflo)

For $w \in W(\mathfrak{g}, \mathfrak{h})$ *let*

$$L_w = \text{unique irreducible quotient of } M(-w\rho)$$

Then

$$\varphi : W \to Prim(\mathfrak{g})_\rho : w \to Ann(L_w)$$

is a surjection.

Parameterizing $Prim(\mathfrak{g})_\rho$ is tantamount to understanding the fiber of $\varphi : W \to Prim(\mathfrak{g})_\rho$

Definition

Let $\sim$ be the equivalence relation on W defined by

$$w \sim w' \quad \Longleftrightarrow \quad Ann(L_w) = Ann(L_{w'})$$

The corresponding equivalence classes of elements of W are called **left cells** in W.

Definition

Let $\approx$ be the equivalence relation on W defined by

$$w \approx w' \quad \Longleftrightarrow \quad \mathcal{O}_{Ann(L_w)} = \mathcal{O}_{Ann(L_{w'})}$$

The corresponding equivalence classes of elements of W are **double cells** in W.

Connection with Weyl group reps

Definition

Fix a finite-dimensional representation σ of W. The $w \in W$ such

$$\mathbb{C}\left\langle W \cdot p_w \right\rangle \approx \sigma$$

comprise the **double cell** in W corresponding to $\sigma \in \widehat{W}$. The representations of W that arise in this fashion are called **special representations** of W.

Theorem

If w, w' belong to same double cell corresponding to a special representation $\sigma \in \widehat{W}$. Then

- *$Ann\left(L_w\right)$ and $Ann\left(L_{w'}\right)$ share the same associated variety.*
- *The unique dense orbit in $AV(Ann(L_w))$ is a special nilpotent orbit $\mathcal{O}$ and the W-rep attached $\mathcal{O}$ by the Springer correspondence coincides with σ.*

Theorem

Let $C \subset W$ be a double cell and let $\sigma \in \widehat{W}$ be the assoc (special) W rep. Then

$$Card\left\{Ann(L_w \mid w \in C\right\} = \dim \sigma$$

Two pictures at inf char ρ

HW-modules

W	$\{L_w \mid w \in W\}$	same inf char
$\cup$	$\cup$	
$\mathcal{C}$: *dbl cell*	$\{L_w \mid w \in \mathcal{C}\}$	same nilpotent orbit
$\cup$	$\cup$	
ℓ : *left cell*	$\{L_w \mid w \in \ell\}$	same primitive ideal

HC-modules

B : block of HC-modules	$\{\pi_x \mid x \in B\}$	same inf char
$\cup$	$\cup$	
C : cell of HC-modules	$\{\pi_x \mid x \in C\}$	same nilpotent orbit
$\cup$	$\cup$	
?	$\{\pi_x \mid x \in ?\}$	same primitive ideal

$L_w := L(-w\rho)$: simple HWM of highest weight $-w\rho - \rho$

$I_w := Ann\,(L_w)$

- I_{w_o} : unique max ideal (augmentation ideal, annihilator of triv rep)
- I_e = unique min PI at inf char ρ ($\leq$ by inclusion)
- I_{s_α}, $\alpha \in \Pi$: "pen-minimal" ideals

Theorem

The primitive ideals I_{s_α}, $\alpha \in \Pi$, are all distinct from each other and I_e. Any primitive ideal strictly containing I_e contains at least one of the I_{s_α}.

Definition

The **tau invariant** of a primitive ideal I containing I_e is

$$\tau(I) = \{\alpha \in \Pi \mid I_{s_\alpha} \subset I\}$$

Connection with W-graphs of HC-modules

Theorem

Let x be an element of a cell C of HC modules and let $\tau(x)$ be its descent set (from W-graph of C). Then

$$\tau(x) = \text{tau-invariant of } Ann(x)$$

A partitioning of HC cells

W-graph of cell: for each element $i \in C$ we attach

- a vertex $v[i]$
- a tau invariant $\tau[i] =$ tau invariant of $Ann(\pi_i)$
- a list of edges with multiplicities $e[i] = [(j_1, m_1), (j_2, m_d), \ldots, (j_k, m_k)]$

τ_0 **subcells**:

$$x \sim_{\tau_0} y \iff \tau(x) = \tau(y)$$
$$C = \coprod_{[x]_0 \in C/\sim_{\tau_0}} [x]_0$$

(Collecting together reps with common assoc variety and common tau invariant)

A partitioning of cells, cont'd

τ_1 **subcells**: Set $\tau_1(x) = \{\tau(y) \mid x \to y \text{ is an edge}\}$

$$x \sim_{\tau_1} y \quad \Longleftrightarrow \quad \tau(x) = \tau(y) \text{ and } \tau_1(x) = \tau_1(y)$$

$$C = \coprod_{[x]_1 \in C/\sim_{\tau_1}} [x]_1$$

τ_2 **subcells**: Set $\tau_2(x) = \{\tau_1(y) \mid x \to y \text{ is an edge}\}$

$$x \sim_{\tau_2} y \quad \Longleftrightarrow \quad \tau_0(x) = \tau_0(y), \tau_1(x) = \tau_1(y), \tau_2(x) = \tau_2(y)$$

$$C = \coprod_{[x]_2 \in C/\sim_{\tau_2}} [x]_2$$

$\vdots$ etc.

$$x \sim_{\tau_i} y \quad \Longleftrightarrow \tau_0(x) = \tau_0(y), \ldots, \tau_i(x) = \tau_i(y) \text{ and}$$

$$C = \coprod_{[x]_i \in C/\sim_{\tau_i}} [x]_i$$

τ_∞ **subcells**: final stable partitioning : $C = \coprod_{[x]_\infty \in C/\sim_\infty} [x]_{\tau_\infty}$

Lemma

The τ_∞ partitioning of a cell of HC-modules is compatible with the partitioning of the cell into subcells consisting of representations with the same primitive ideal:

$$Ann(x) = Ann(y) \quad \Longrightarrow \quad x \text{ and } y \text{ live in same } \tau_\infty\text{-subcell.}$$

(follows from well-definedness of Translation Functor for primitive ideals)

Theorem

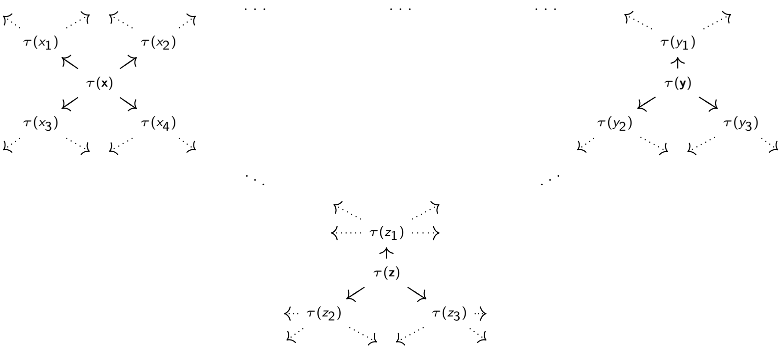
Let C be any cell in any real form of any exceptional group G. Then the τ_∞ partitioning of C coincides precisely with the partitioning of the cell into sets of irr HC modules sharing the same primitive ideal:

$$x \sim_\infty y \quad \Longleftrightarrow \quad Ann(x) = Ann(y)$$

proof:

$$\#P_\infty\text{-subcells} = \text{dim special } W\text{-rep attached to cell} = max\#\text{primitive ideals in cell}$$

Cell W-graph



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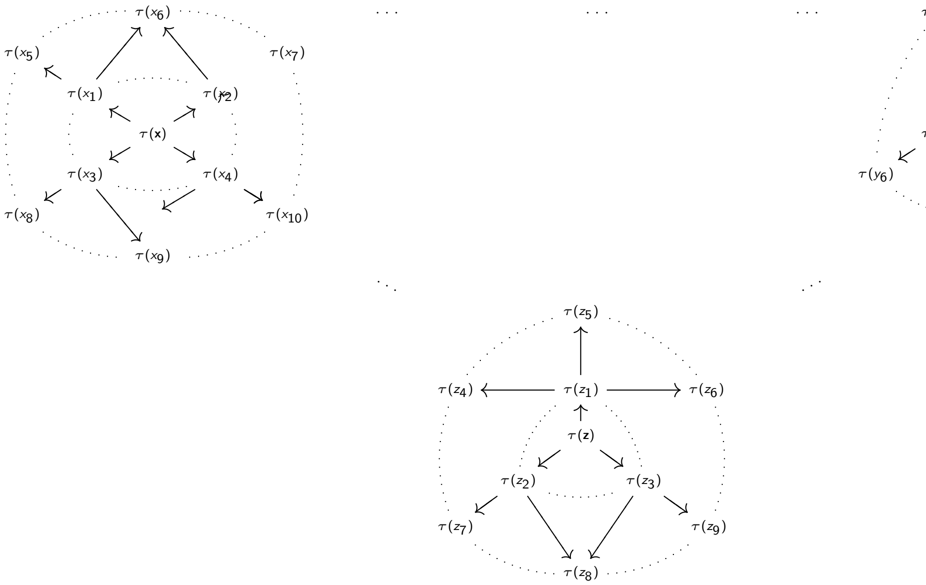
$\tau(x_6)$
$\tau(x_5)$
$\tau(x_7)$
$\tau(x_1)$
$\tau(x_2)$
$\tau(\mathbf{x})$
$\tau(x_3)$
$\tau(x_4)$
$\tau(x_8)$
$\tau(x_{10})$
$\tau(x_9)$
$\tau(y_6)$
$\tau(z_5)$
$\tau(z_4)$
$\tau(z_1)$
$\tau(z_6)$
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